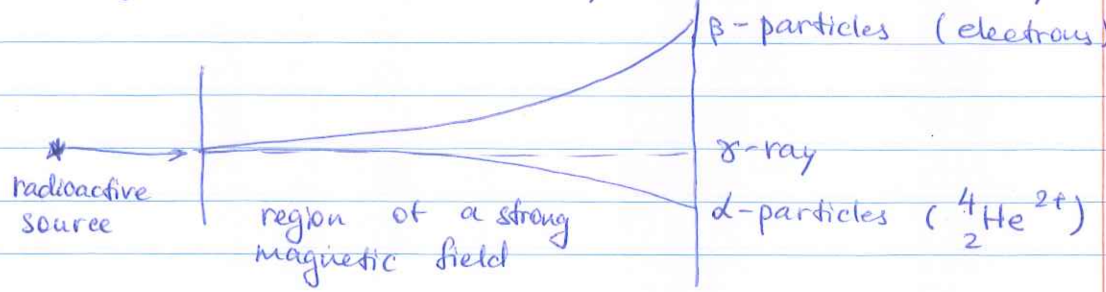
Radioactivity and nuclear reactions

Natural radioactivity

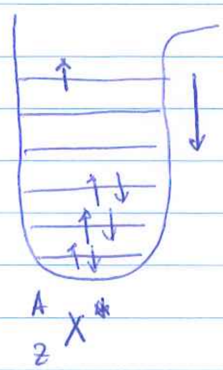
Heavy nuclei can spontaneously decay, emitting various products.
Radioactive elements that start the decay series
$^{238}_{92}U$, $^{235}_{92}U$, $^{232}_{90}Th$, $^{237}_{93}Np$

Average lifetime 10^9–10^{10} yr (except for Np – 10^6 yr)



Can we explain these different outcomes?

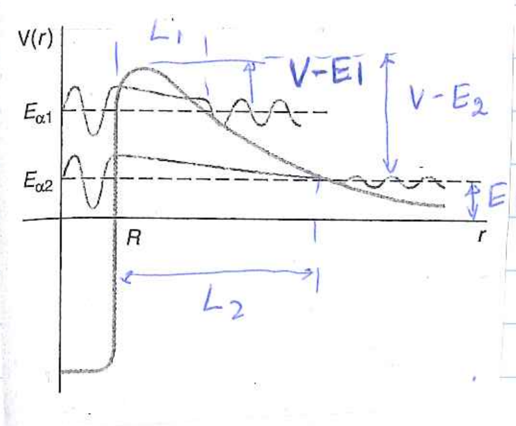
<u>γ-rays:</u> emitted when a nucleus transitions b/w various energy levels)

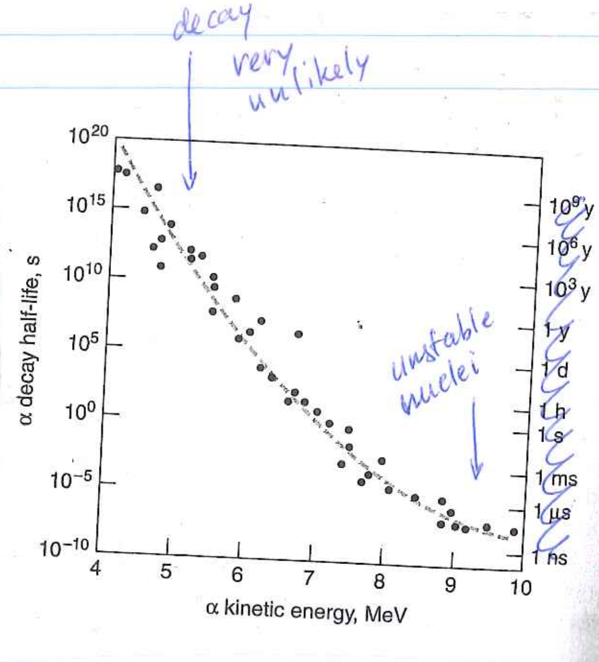


when a nucleons returns to its ground state, the excess energy is relised in form of a highly energetic photon
γ - radiation
(energy range – 1MeV ÷ 1GeV)

a nucleus in the excited state

γ-ray spectrum can be used to identify nuclius, just as optical spectroscopy





Tunneling probability

$$T \propto \exp\left[-2L\sqrt{\frac{2m(V-E)}{\hbar}}\right]$$

L — length of the barrier

$\sqrt{\frac{2m(V-E)}{\hbar}}$ — α

$(V-E)$ — hight of the barrier, detemines disintegration energy

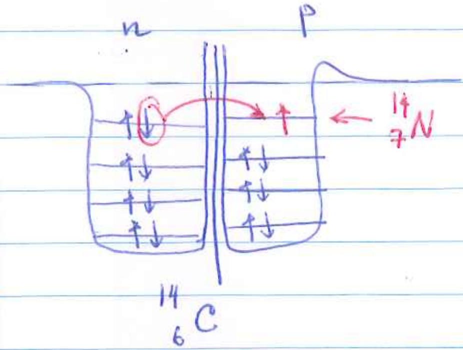
Depending on the energy ~~of the~~ level of α-particle "inside" the nucleus, its final disintegration energy will be different. The closer the level is to the top of the barrier, the easier it is for the particle to tunnel out, ~~but~~ and its ~~energy~~ kinetic energy will (lower) be ~~low~~. high. To produce a particle with ~~higher~~ kinetic energy, the α particle must tunnel from a lower energy level, but its probability will be much lower.

β - decay

$${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^- + \bar{\nu}$$

one neutron inside the nucleus turns into a proton. Such transformation is enabled by a weak force. Reverse process is also possible: $n \rightarrow p$

$${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + e^+ + \nu$$



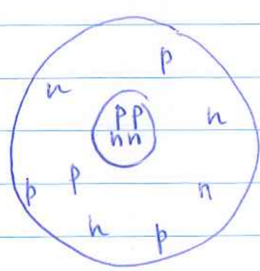
$${}^{14}_{6}C \rightarrow {}^{14}_{7}N + e^- + \bar{\nu}$$

more balanced number of n and p

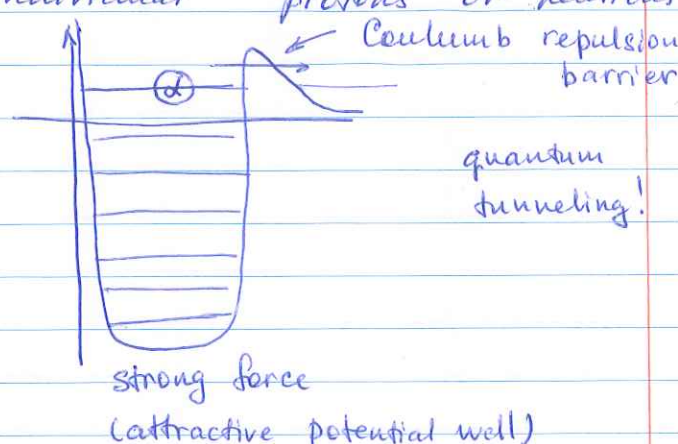
α- decay

$${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + \underbrace{{}^{4}_{2}He}_{\alpha\text{-particle}}$$

α- particles are so stable, that it is more energetically efficient to emit an α-particle than to emit individual protons or neutrons



Heavy nucleus



How can we figure out if the decay process is possible?

Conservation laws!

- nucleon number conservation
- energy conservation
- momentum / angular momentum conservation

α- decay $\quad {}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$

Initial energy: $M_X c^2$

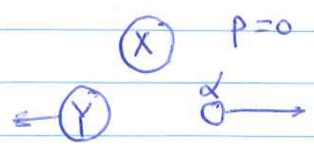
Final energy: $M_Y c^2 + M_\alpha c^2 + Q$ ← disintegration energy

$$Q = (M_X - M_Y - M_\alpha)c^2$$

In general, the disintegration energy is the mass difference b/w the initial and final products

Can all this energy be transferred to the α-particle?

No - ~~energy cons~~ momentum conservation

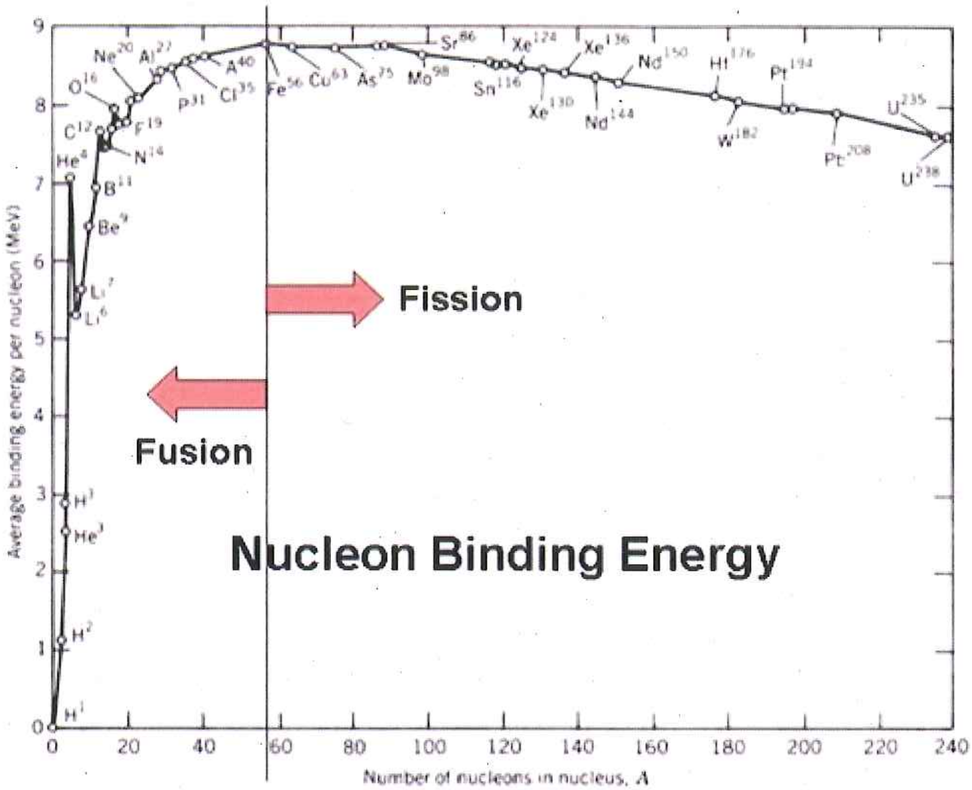


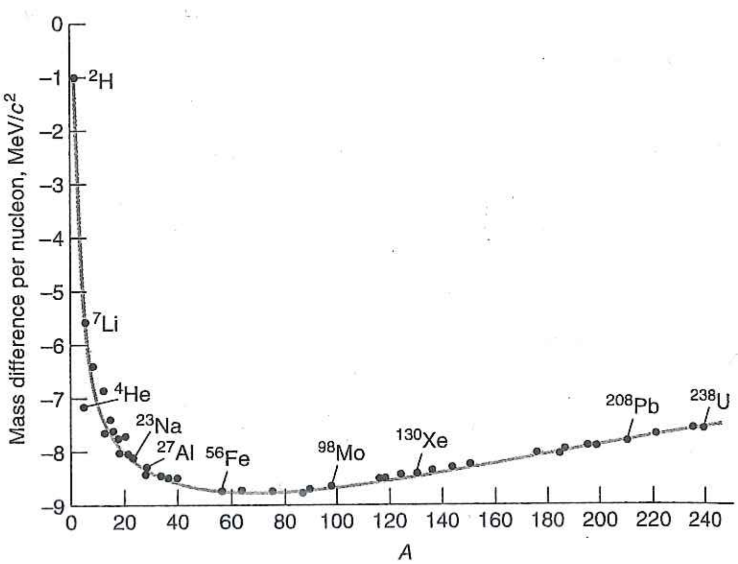
$|P_Y| = |P_\alpha| \qquad K_Y/M_Y = K_\alpha/M_\alpha$

$$Q = K_\alpha + K_Y = K_\alpha + \frac{M_\alpha}{M_Y} K_\alpha$$

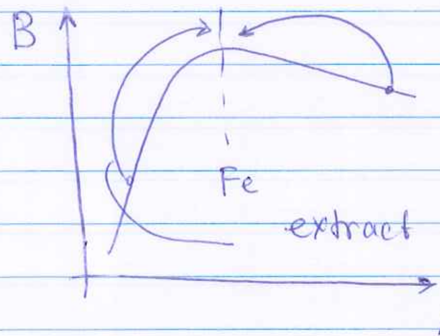
$$K_\alpha = \frac{M_Y}{M_Y + M_\alpha} Q$$

Interestingly, if one repeats the calculations for ^{238}U emitting just one nucleon via tunneling, ~~it~~ the mass of the end products will exceed the mass of the original nucleus, so this process won't happen.


Fission
Fusion
Nucleon Binding Energy
Average binding energy per nucleon (MeV)
Number of nucleons in nucleus, A


Mass difference per nucleon, MeV/c²
A
²H
⁷Li
⁴He
²³Na
²⁷Al
⁵⁶Fe
⁹⁸Mo
¹³⁰Xe
²⁰⁸Pb
²³⁸U

Nuclear reactions



extract energy by splitting a heavy nucleus (fission)

extract energy by merging the light nuclei (fusion)

Nuclear fission

$^{1}_{0}n + ^{235}_{92}U \longrightarrow \underbrace{X + Y}_{\text{daughter nuclei}} + \text{neutrons}$

for example: $^{1}_{0}n + ^{235}_{92}U \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3\,^{1}_{0}n$
energy emitted – 176 MeV

Self - sustaining process (chain reaction)
produced neutrons cause exponentially growing number of splitting U atoms

Nuclear fusion

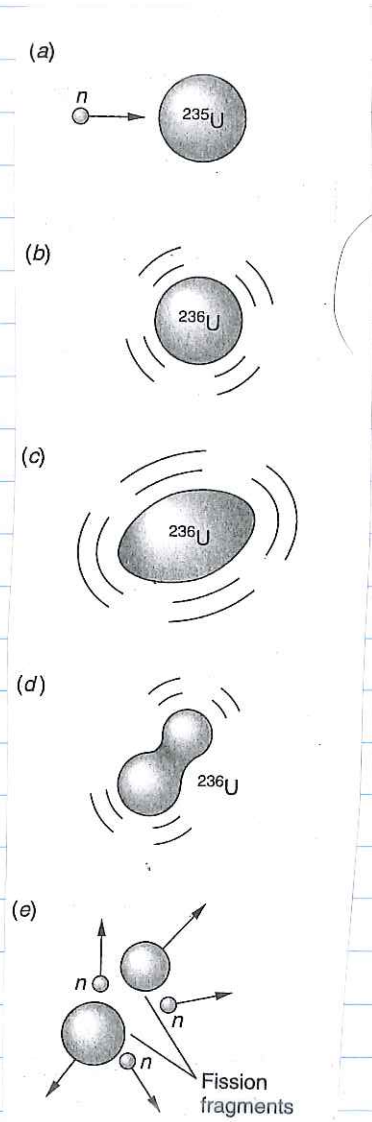
~~p+~~ $p + p \rightarrow ^{2}_{1}D + \text{stuff}$

$^{2}_{1}D + p \rightarrow ^{3}_{2}He$

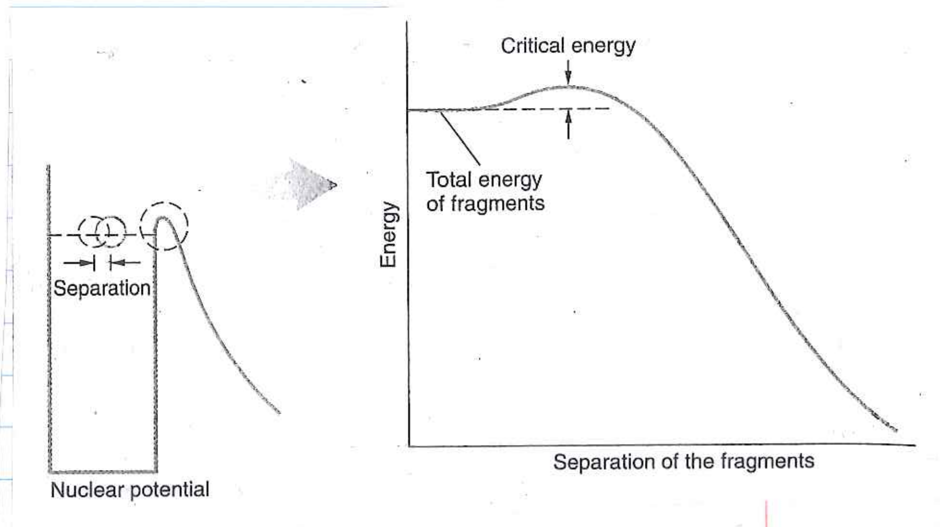
$p + ^{3}_{2}He \rightarrow ^{4}_{2}He + \text{stuff}$ or $^{3}_{2}He + ^{3}_{2}He \rightarrow ^{4}_{2}He + p + p$
energy produced ~ 24 MeV

Main challenge: bring particles close enough to overcome coulumb repulsion
- High kinetic energy ($\sim 10^7 K$) → Sun
- Strong plasma confinement (not sure how to build, yet)

Steps of ~~the~~ nuclear reaction



$$n + {}^{235}U \rightarrow {}^{236}U \quad (+\gamma)$$



$$n + {}^{235}U$$
$$\downarrow$$
$${}^{236}U \rightarrow {}^{92}Kr + {}^{142}Ba + 2n + 179.4\,MeV$$