Quantum theory of light

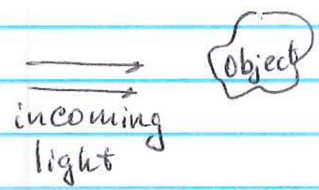
Maxwell's equations, describing light as electro-magnetic wave, appeared to resolve the ~~question~~ ~~th~~ wave-particle discussion once and for all
Light is a Wave

This new understanding induce a lot of interest in e-m radiation and its interaction with matter. And as the new experimental findings start to pile up, some strange descrepancies showed up.

- One of the "dark clouds" - ~~photoelectric~~ unexplainable behavior of a black body radiation,
- Photo electric effect
- X-ray diffraction and Compton effect

All these effects required carpuscular treatment of light to be understood.

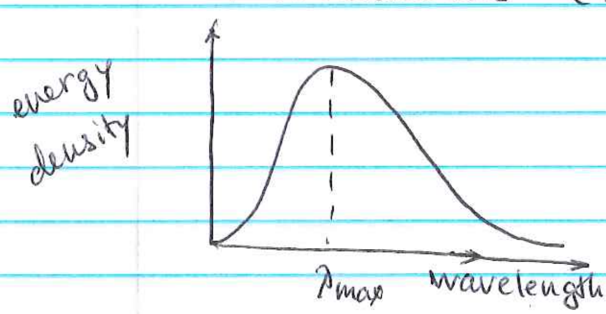
Black body radiation



Incoming light can be
- transmitted (no interaction)
- reflected or scattered
- absorbed

Black body is a perfect absorber - it absorbs radiation of all wavelengths that falls on it
However, BB must also emit energy if it is in a thermal equilibrium. The frequencies of the emitted radiation are independent on the parameters of the original, absorbed, radiation, but instead depend only on the <u>temperature</u> of BB

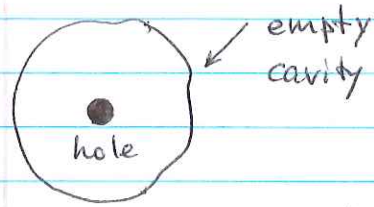
Stephan-Boltzmann law $P_{total} = \sigma T^4$
Can be derived using Maxwell's equations and thermodinamics (Boltzmann statistics)



Wein's law: $\lambda_{max} \cdot T = const$

Classical physics had a lot of difficulties explaining this energy spectrum

Typical model of a black body



A hole is a perfect absorber: all incoming radiation goes in; then it is absorbed and reemitted many times inside the cavity, so whatever is radiated back is in thermal equilibrium with the cavity.

Number of possible radiation modes ~~with~~ (different standing waves) inside a cavity around some ~~Ad~~ frequency f

$$\Delta N(f, f+\Delta f) = \underbrace{N(f)}_{\text{mode density}} \Delta f = \frac{8\pi}{c^3} V f^2 \Delta f$$

Rayleigh-Jeans formula:
each mode has equal amount of energy $k_B T$
Thus, the ~~ener~~ radiated spectral energy density
(energy emitted per unit area per unit ~~fr~~ frequency

$$u_{RJ}(f,t) = \frac{8\pi}{c^3} \cancel{V} \cdot f^2 \cdot k_B T \quad \propto f^2 \cdot T$$

Problem $u_{RJ} \xrightarrow{f \to \infty} \infty$ ultraviolet catastroph!

Wein's exponential (empirical) law

$$u_W(f,t) = A f^3 e^{-\beta f / T} \qquad A, \beta \text{ – universal constants}$$

works reasonably well for high frequencies, but not for low frequencies.

Plank's resolution — more general equation

$$u(f,T) = \frac{8\pi h f^3}{c^3} \left(\frac{1}{e^{\frac{hf}{k_B T}} - 1} \right) \qquad h = 2\pi \cdot 10^{-34}\ J \cdot s$$

For $hf \gg k_B T$ (high-frequency limit)

$$u(f,T) = \frac{8\pi h f^3}{c^3} e^{-hf/k_B T} \qquad \text{Wein's formula}$$

For $hf \ll k_B T$ $\qquad e^{\frac{hf}{k_B T}} \approx 1 + \frac{hf}{k_B T}$

$$u(f,T) = \frac{8\pi k_B T \cdot f^2}{c^3} \qquad \text{Rayleigh - Jeans formula}$$

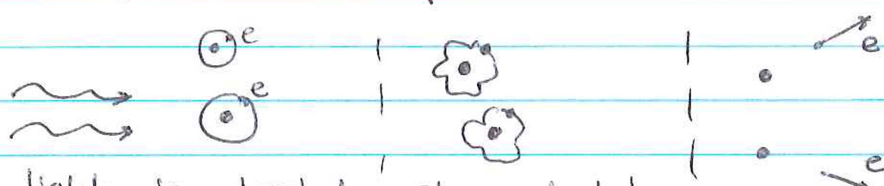
However, the only way Plank could derive this expression was when he assumed that energy can be emitted in "chunks" of hf, so that high-frequency modes, for which $k_B T \ll hf$ are not emitted at all (UV-cut-off)

Plank first considered such "fix" unphysical, and worked hard for 6 years to find a "more proper" solution

Photoelectric effect (first observed by Hertz during his tests of Maxwell equations, studied more carefuly by Lenard in 1900-1902)

High-energy light leads to the emission of charged particles (electrons) from the metal surface.

Classical description



light is absorbed by atoms | atoms start to vibrate | electrons are "shaken off"

Expectation: the more intense the light is, the ~~more~~ faster electrons are emitted.

Observation: 1) Energy of emitted electrons is independent of the light intensity, and depends only on light frequency.

2) Light intensity changes only the number of ~~photon~~ electrons, not their energy.

Einstein pointed out that one can explain these observations, if one adopts the carpuscular model of light, in which the light beam is a stream of particles carrying the energy (hf)

Then an atom absorbs this quanta of energy, that defines the kinetic energy of $\bar{e}$

$$hf = K_e + \varphi$$

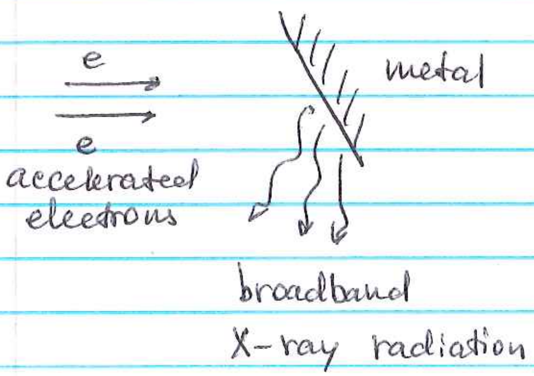
work function, binding energy of $\bar{e}$ inside the metal

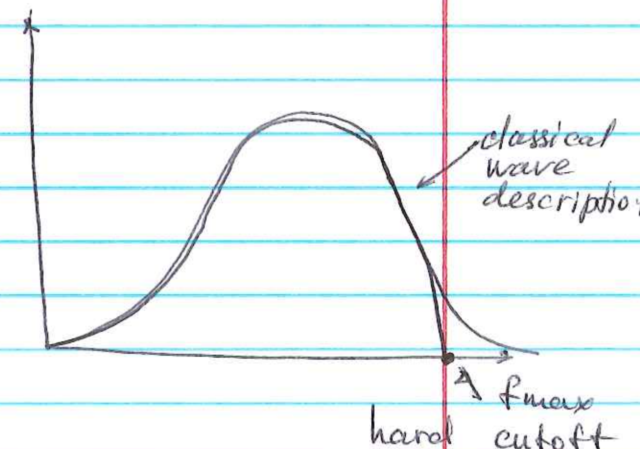
At the same time, special relativity predicted the existance of massless particles $E = c \cdot p$
One can test it using high-energy radiation

X-rays – e-m radiation with $\lambda = 0.01 - 10\ nm$ $(10^{-11} - 10^{-8} m)$
discovered by Roentgen in 1895.

Their wave nature was demonstrated only later by using a periodic crystal structure as a diffraction grating
(Currently, X-ray diffraction is a standard diagnostic tool for material science)
However, two phenomena required quantum description:

1. Brehmstrasse cutoff frequency



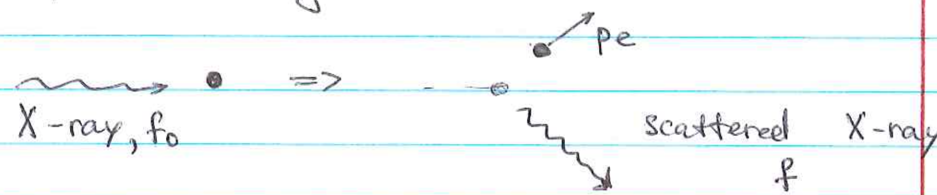


$$h f_{max} = K_e$$

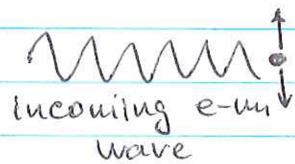
The maximum emitted frequency was limited by a kinetic energy of a single electron.

Compton effect : ~~electro~~

X-ray scattering on an electron



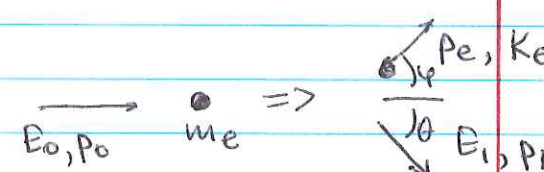
Classical wave description



electron oscillates with frequency f_0 in electric field, accelerating charge emits light, ~~so~~ proportional to its acceleration. Thus, any re-emitted radiation must have the same frequency as an incoming wave!

Not what was observed

Quantum treatment



Energy conservation! $E_0 + \cancel{m_e c^2} = E_1 + \cancel{m_e c^2} + K_e$

$$E_0 - E_1 = K_e$$

If $E_{photon} = hf$, $hf_0 - hf_1 = K_e \Rightarrow f_0 \neq f_1$!

Momentum conservation: $\begin{cases} p_0 = p_e \cos\varphi + p_1 \cos\theta \\ 0 = p_e \sin\varphi - p_1 \sin\theta \end{cases}$

$$p_e^2 = p_0^2 - 2p_0 p_1 \cos\theta + p_1^2$$

$E_0 = cp_0$, $E_1 = cp_1$, $E_e = \sqrt{(p_e c)^2 + (m_e c^2)^2} = m_e c^2 + K$

$$p_e^2 c^2 = K^2 + 2K m_e c^2$$

Bringing all these together

$$m_0 c (p_0 - p_1) = p_0 p_1 (1 - \cos\theta)$$

$$\left(\frac{1}{p_1} - \frac{1}{p_0}\right) = \frac{1}{m_e c}(1 - \cos\theta) \qquad p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$

$$\lambda_1 - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta) = \lambda_c (1 - \cos\theta)$$

For an electron $\lambda_c = 2.43 \cdot 10^{-12}$ m