Relativistic Doppler effect

(describes frequency change of an EM wave if a source (or detector) is moving

Brief reminder: light is an electro-magnetic wave

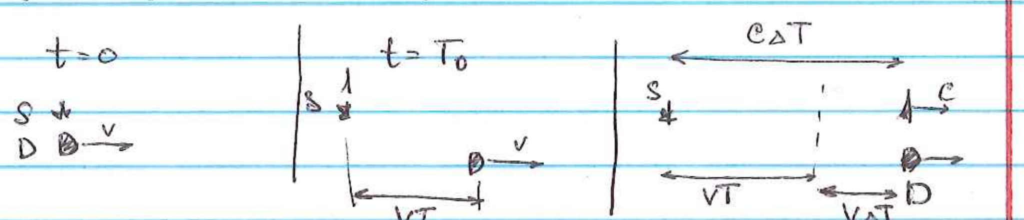
$$E = E_0 \cos\left(\frac{2\pi}{\lambda}\cdot x - 2\pi f\cdot t\right) \quad \text{<in vacuum>}$$

f - frequency, λ - wavelength, $f = c/\lambda$

Light clocks: let's assume that every time the light pulse hits the bottom mirror, a flash of light is emitted, to mark each "tick." (duration T_0)

If a detector moves away from the source (clock) how much time will pass between each tick?

Let's for simplicity assume that at $t=0$ the source and the detector are collocated



$c\Delta T = V(T_0 + \Delta T)$

$\Delta T = T_0 \frac{V}{c-V}$

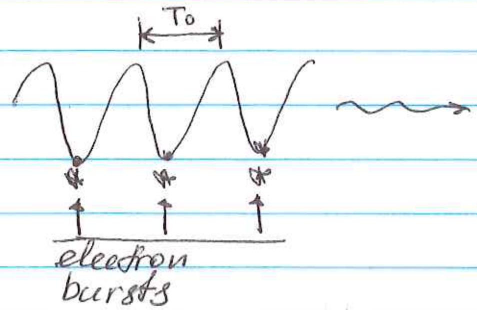
In the RF of the source the time b/w two detected ticks is $T = T_0 \cdot \frac{V}{c-V} + T_0 = T_0 \frac{c}{c-V}$

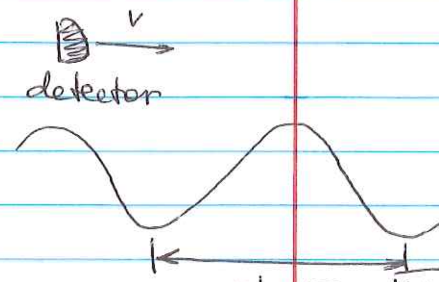
In the RF of the moving detector, however, the time is dilated, so it perceive the ticks coming in time intervals

$$T' = \frac{T}{\gamma} = \frac{1}{\gamma} T_0 \frac{c}{c-V} = T_0\sqrt{(1-V^2/c^2)}\ \frac{1}{1-V/c} = T_0\sqrt{\frac{1+V/c}{1-V/c}}$$

Detector sees ticks coming closer to each other if the detector approaches the source (or vice versa), and they become ~~more~~ longer if the detector is receding

We can use the same logic to deduce the relativistic changes in EM frequency





$T' = T_0 \sqrt{\frac{1+v/c}{1-v/c}}$

Frequency of the source $f_0 = \frac{1}{T_0}$

Frequency ~~of the~~ measured by the detector

$$f = \frac{1}{T'} = \frac{1}{T_0}\sqrt{\frac{1-v/c}{1+v/c}} = f_0\sqrt{\frac{1-v/c}{1+v/c}}$$

If the source and the detector are moving away $f = f_0\sqrt{\frac{1-v/c}{1+v/c}} < f_0$ $\quad \lambda_0 = \frac{c}{f_0}$; $\lambda = \lambda_0\sqrt{\frac{1+v/c}{1-v/c}} > \lambda_0$

The detected light is <u>red-shifted</u>

If the detector is approaching $v \to -v$

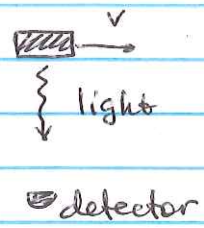
$$f = f_0\sqrt{\frac{1+v/c}{1-v/c}} > f_0 \quad \text{or} \quad \lambda = \lambda_0\sqrt{\frac{1-v/c}{1+v/c}} < \lambda_0$$

the detected light is <u>blue-shifted</u>

(Joke: red traffic will appear green if you approach it fast enough)

Since the universe is expanding, distant galaxies move away from us, so their light is <u>red-shifted</u>.

Transverse Doppler effect



No correction for the source motion, still time dilation

$$f'_{transverse} = \frac{1}{\gamma} f_0 = \sqrt{1 - v^2/c^2} \cdot f_0$$

How big is the Doppler effect ~~is~~ if the source/detector relative velocity is not close to c?

If $v/c \ll 1$, we can use Taylor expansion to find the ~~corr~~ first order correction to the frequency

$$(1+x)^{\alpha} \approx 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 \ldots \quad \text{if } x \ll 1$$

if $\alpha = \pm 1/2$

$$(1 \pm x)^{1/2} \approx 1 \pm \frac{x}{2} \qquad (1 \pm x)^{-1/2} \approx 1 \mp \frac{x}{2}.$$

$$\sqrt{\frac{1 - v/c}{1 + v/c}} \approx \left(1 - \frac{1}{2}\frac{v}{c}\right)\left(1 - \frac{1}{2}\frac{v}{c}\right) \approx 1 - \frac{v}{c} \quad \left(\text{or } 1 + \frac{v}{c} \text{ for } v \to -v\right)$$

$$f = f_0 \sqrt{\frac{1 - v/c}{1 + v/c}} \approx f_0 - \underbrace{f_0 \frac{v}{c}}$$

$$f = f_0 \sqrt{\frac{1 + v/c}{1 - v/c}} \approx \overbrace{f_0 + f_0 \frac{v}{c}}$$

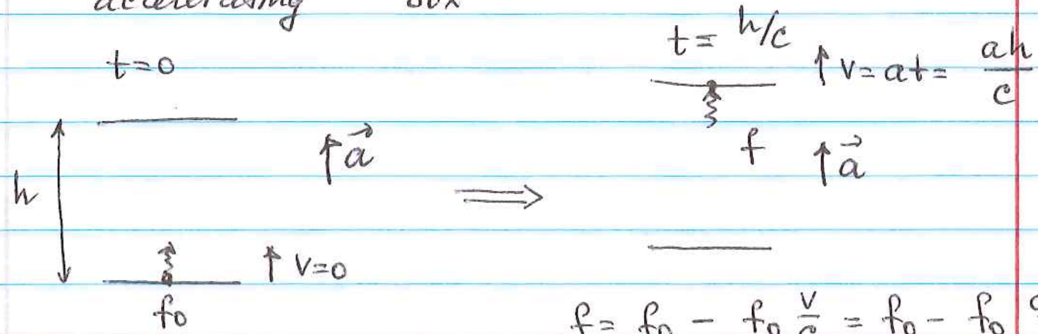
Doppler shift positive for approaching sources, negative for receding.

But for the transverse Doppler effect

$$f'_{transv} = \frac{f_0}{\gamma} = \sqrt{1 - v^2/c^2}\, f_0 \approx f_0 - \underbrace{\frac{v^2}{c^2} f_0}_{\text{by a factor of } v/c \ll 1.}$$

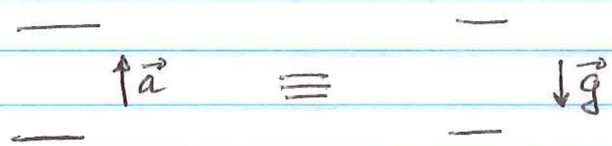
Gravitational red shift

accelerating box



$$f = f_0 - f_0 \frac{v}{c} = f_0 - f_0 \frac{ah}{c^2}$$

Equivalence Principle: there is no experiment done in a small confined space which can tell the difference between a uniform gravitational field and an equivalent acceleration



Gravitational red shift $\Delta f = - f_0 \frac{gh}{c^2}$