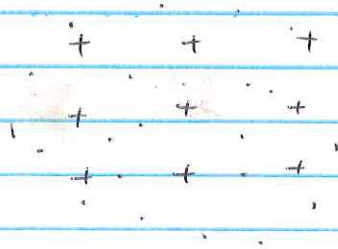
Conductivity

One requires free-moving electrons to conduct electrical current

Metallic bond: ionic crystal structure + free shared electrons



electron gas: electrons move as free particles b/w lattice ions (semiclassical Drude model)

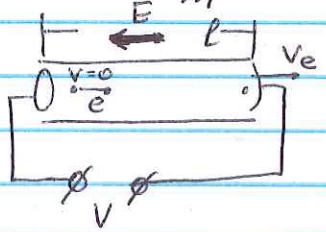
Ohm's law: $V = IR$ (bulk) or $\vec{j} = \sigma \vec{E}$ (local)

where $\sigma = e/\rho A$ conductivity ($I = jA$, $V = E \cdot \ell$)

Contraversy!

If an electron is free to move, they are accelerated by the external electric field

$a = \frac{eE}{m}$; $v_e^2 = 2a \cdot \ell = \frac{2e\ell}{m} E$



But current density $j = n \cdot e \cdot v_e$

$j^2 \propto E$ ... what is wrong?

Electrons do not move unpretarbed from one end of the conductor to another, their acceleration is constantly interrupted by collisions. Electron move randomly with relatively high speed, and external electric field provides relatively small common drift $v_{drift} = a \cdot \tau$ where τ is the time b/w collisions ($v_{drift} \approx 1 mm/s$)

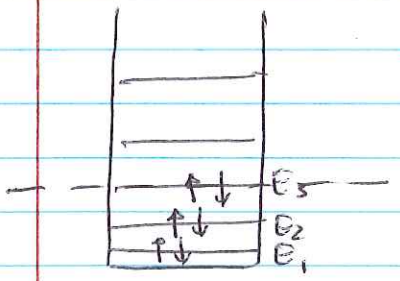
Main source of collisions - imperfections in the lattice, not the regular ions.

Quantum theory of solids

Classical Drude theory is quite successful in explaining electrical and heat conductivity of metals. However, it predicts incorrect temperature dependence, when it assumes that electrons are moving with average $v_{rms} = \sqrt{\frac{8 k T}{3m}}$

What is the energy of the electrons, We must remember that electrons are fermions. That means that even at zero temperature ($T = 0k$) their total energy cannot be zero

Bound states (1D case)



6 electrons

$E_{min} = 2E_1 + 2E_2 + 2E_3$

Highest occupied state $- E_3$

Fermi energy

Free electrons — does Fermi energy exist?

Yes, since we must respect the uncertainty principle

$\Delta x \cdot \Delta p_x \sim h = 2\pi\hbar$ $\quad \Delta x \sim 2\pi\hbar / \Delta p_x$

All electrons must have energies & positions resolved beyond the uncertainty principle

How many distinguishable electrons one must have in volume V -?

$$\Delta N_{(p)} = 2 \frac{V}{\Delta x \cdot \Delta y \cdot \Delta z} = 2 \frac{V}{(2\pi\hbar)^3} \underbrace{\Delta p_x \Delta p_y \Delta p_z}_{d^3 \vec{p}}$$

More precisely, the number of electrons can be connected to the electrons Fermi energy

$$N = \frac{2V}{(2\pi\hbar)^3}\int_0^{P_F} d^3\vec{p} = \frac{V}{\pi^2\hbar^3}P_f^3 \qquad E_F = \frac{P_f^2}{2m}$$

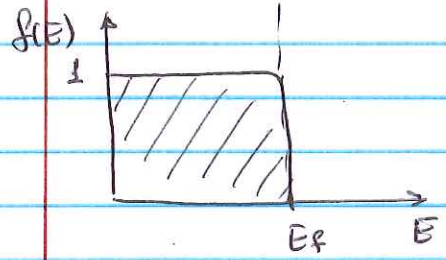
Fermi energy $$E_F = \left(3\pi^2\frac{N}{V}\right)^{2/3}\frac{\hbar^2}{2m}$$

Fermi energy very weakly depends on temperature.

Electron energy distribution: Fermi-Dirac

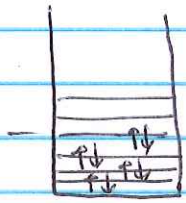
$$f(E) = \frac{dn(E)}{n} = \frac{1}{e^{(E-E_F)/k_BT}+1}$$

$T = 0$

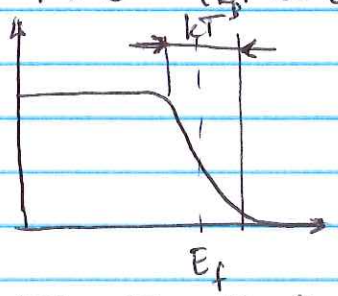


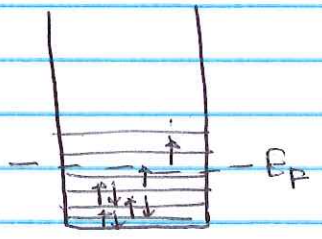
All states $E < E_F$ are filled, all states $E \geq E$ are empty

analog

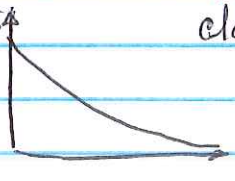


$T > 0 \quad (k_BT \ll E_F)$





If $T \sim E_F/k_B$, Fermi-Dirac approaches Maxwell distribution



classical description is good

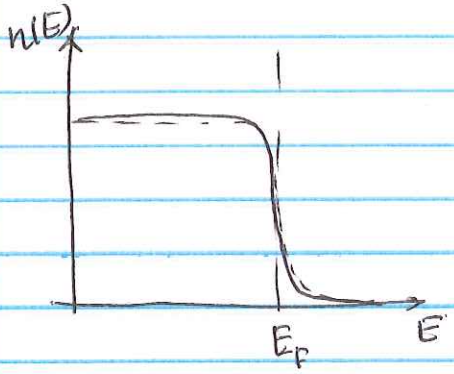
For metals a typical Fermi energy
$E_F \sim$ a few eV (Copper - 7eV)

Majority of electrons move with ~~speed~~ kinetic energies close to $E_F = mv_F^2/2 \Rightarrow v_F = \sqrt{2E_F/m}$
For copper $v_F \simeq 1.6 \cdot 10^6$ m/s
(Faster than classical prediction, and does not depend on temperature.

From experimentally measured conductivity the mean free path between the collisions $l_{mfp} = v_F \cdot \tau \sim 400$ Å
150 x distance b/w Cu ions
(due to impurities)
How come electrons don't bump into the lattice? Because they are waves!

Electrons are not really free gas, but their energy is restricted by the band structure, dictated by the lattice. That also means that the conductance will depend on the band structure.

Fermi - Dirac distribution + band structure



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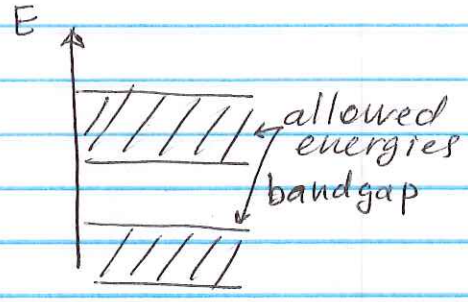
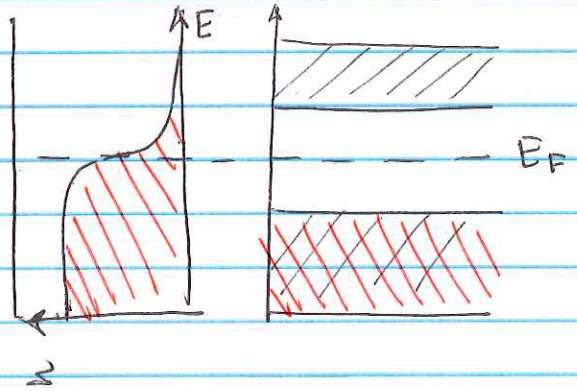


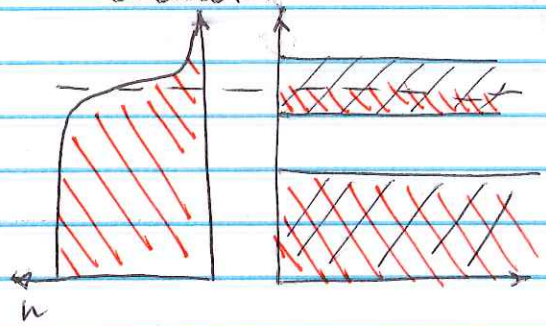
Insulators



empty band

Filled band
no available $\bar{e}$ states

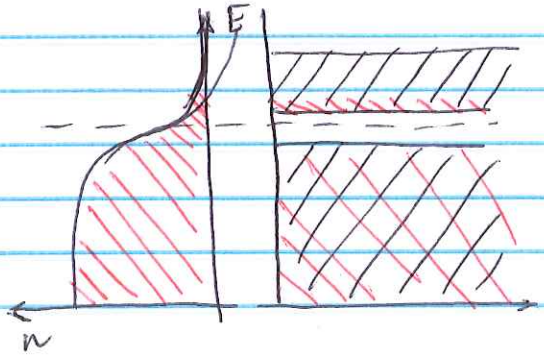
Conductors



Half-filled band - plenty of free states, valence band

Filled band

Semiconductors



some electrons are available in valence band, depending on T