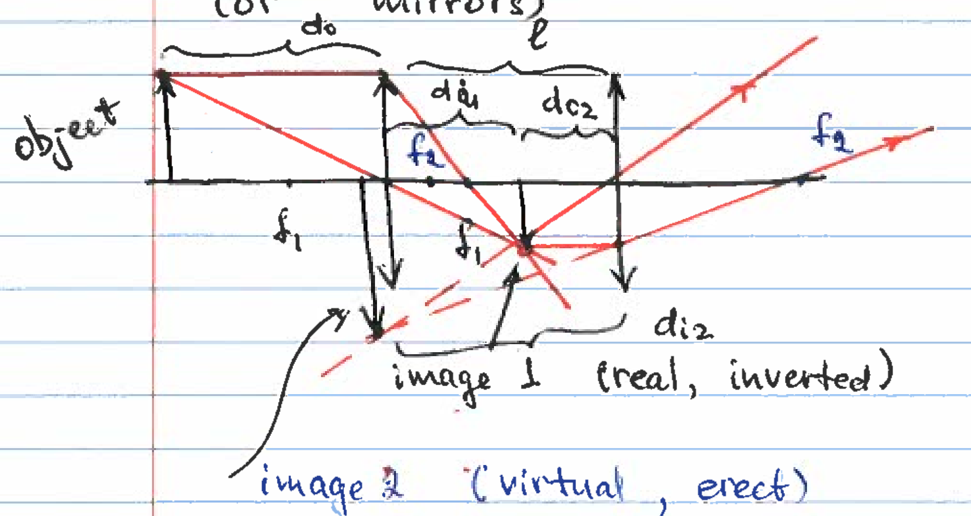
System of lenses

Typically lenses come in only fixed focal distances, so to match application requirements one often needs a combination of lenses (or mirrors)



lens 1

$$\frac{1}{d_o} + \frac{1}{d_{i1}} = \frac{1}{f_1}$$

$$d_{o2} = \ell - d_{i1}$$

$$\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_2}$$

Illustrate with numbers $d_o = 30cm$, $f_1 = 10cm$, $f_2 = 20cm$, $\ell = 20cm$

① $\frac{1}{30} + \frac{1}{d_{i1}} = \frac{1}{10} \Rightarrow \frac{1}{d_{i1}} = \frac{2}{30}$ $\quad d_{i1} = 15cm$ real

② $d_{o2} = 20\ cm - 15cm = 5cm$

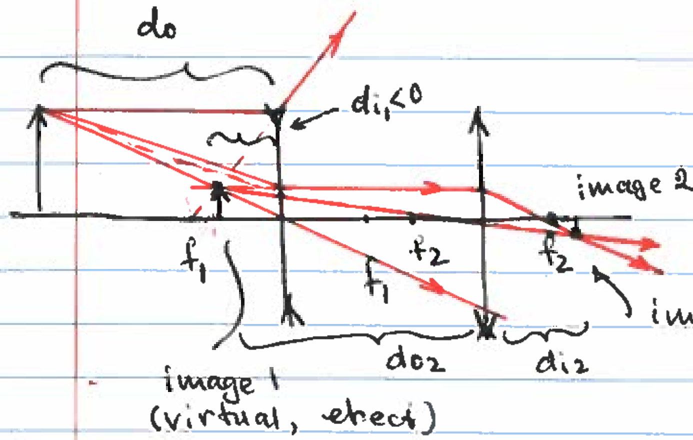
③ $\frac{1}{5cm} + \frac{1}{d_{i2}} = \frac{1}{20cm}$ $\quad \frac{1}{d_{i2}} = -\frac{3}{20cm}$ $\quad d_{i2} = -\frac{20}{3}cm$ virtual

$$M_1 = -\frac{d_{i1}}{d_o} = -\frac{15cm}{30cm} = -\frac{1}{2} \quad \text{(inverted)}$$

$$M_2 = -\frac{d_{i2}}{d_{o2}} = -\frac{-20}{3} \cdot \frac{1}{5} = \frac{4}{3} \quad \text{(erect)}$$

Total magnification $M = M_1 \cdot M_2 = -\frac{1}{2} \cdot \frac{4}{3} = -\frac{2}{3}$ (inverted with respect to the original image)

Example 2: intermediate virtual ~~object~~ image



$d_o = 30\,cm$, $f_1 = -10\,cm$

$f_2 = 20\,cm$, $\ell = 30\,cm$

① $\frac{1}{d_o} + \frac{1}{d_{i_1}} = \frac{1}{f_1}$ $\quad \frac{1}{30} + \frac{1}{d_{i_1}} = -\frac{1}{10} \Rightarrow \frac{1}{d_{i_1}} = -\frac{4}{30}$ $\quad d_{i_1} = -\frac{30}{4}\,cm$

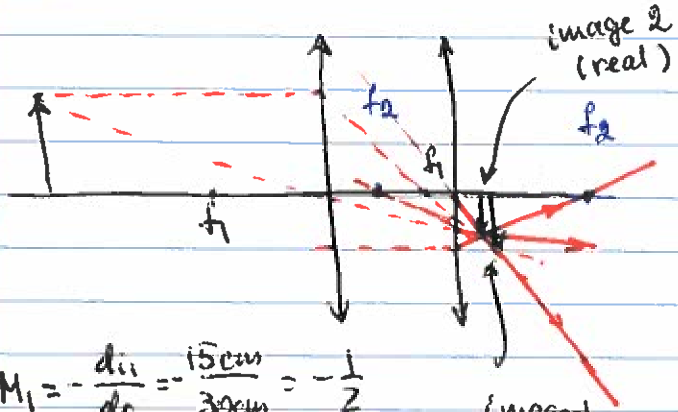
② $d_{o_2} = \ell - d_{i_1} = 30\,cm + \frac{30}{4}\,cm = 37.5\,cm$ (virtual image) (real object)

③ $\frac{1}{d_{o_2}} + \frac{1}{d_{i_2}} = \frac{1}{f_2}$ $\quad \frac{1}{37.5} + \frac{1}{d_{i_2}} = \frac{1}{10}$ $\quad \frac{1}{d_{i_2}} = \frac{1}{10} - \frac{2}{75} = \frac{11}{150}$

$d_{i_2} = \frac{150}{11}\,cm$

real image

Example 3: intermediate virtual object



$d_o = 30\,cm$, $f_1 = 10\,cm$, $f_2 = 20\,cm$

$\ell = 12\,cm$

from the first example

$d_{i_1} = 15\,cm$

$d_{o_2} = 12\,cm - 15\,cm = -3\,cm$

$\frac{1}{d_{o_2}} + \frac{1}{d_{i_2}} = \frac{1}{f_2}$

$-\frac{1}{3} + \frac{1}{d_{i_2}} = \frac{1}{20}$

$\frac{1}{d_{i_2}} = \frac{1}{20} + \frac{1}{3} = \frac{23}{60}$

$d_{i_2} = \frac{60}{23}\,cm$

$M_1 = -\frac{d_{i_1}}{d_o} = -\frac{15\,cm}{30\,cm} = -\frac{1}{2}$

$M_2 = -\frac{d_{o_2}}{d_{o_2}} = -\frac{(-3\,cm)}{(60/23\,cm)}$

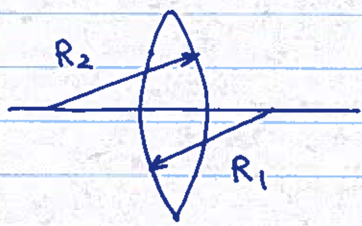
$= \frac{23}{20}$

$M_1 \cdot M_2 = -\frac{23}{40}$ inverted

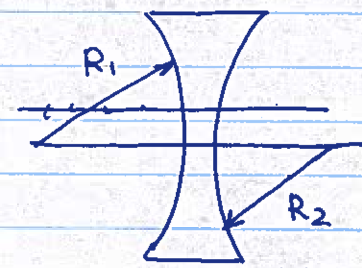
Lens maker equation

A standard lens is a spherical doublet: two spherical surfaces

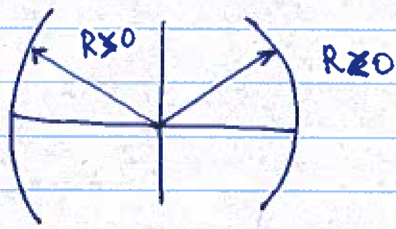
Converging lens



Diverging lens



Radii of curvature: positive if the center of the sphere is on the ~~left~~ right, negative – on the ~~right~~ left



Thin lens: $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Thick lens: $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right)$

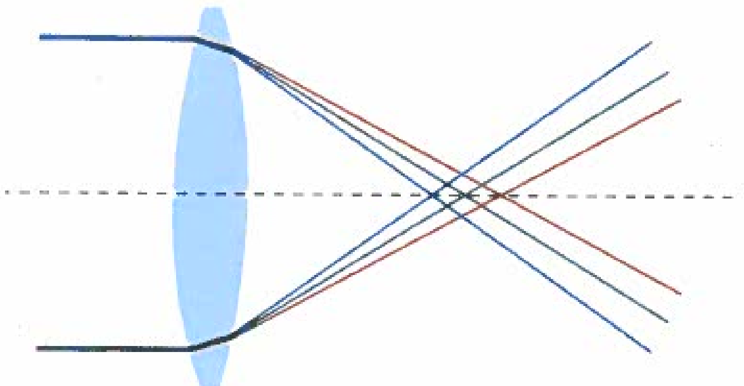
d – thickness of the lens

If $R_1, R_2 \gg d$ → negligeable

Becomes significant if $R_1, R_2 \sim d$

(that is why it is hard to make a lens with very short focus)

Optical Anomalies and Lens Corrections

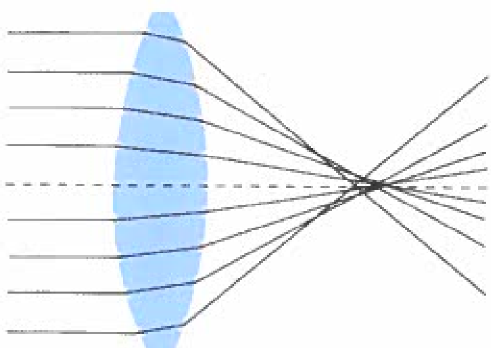


Lensmaker equation

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

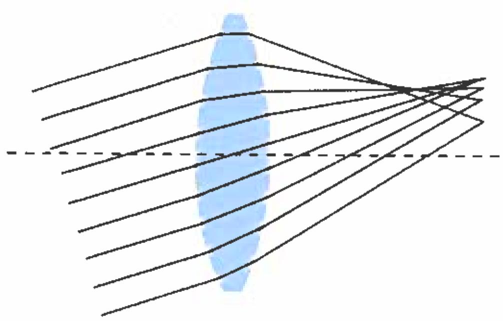
Chromatic aberrations

n value is different for different colors



Spherical aberrations

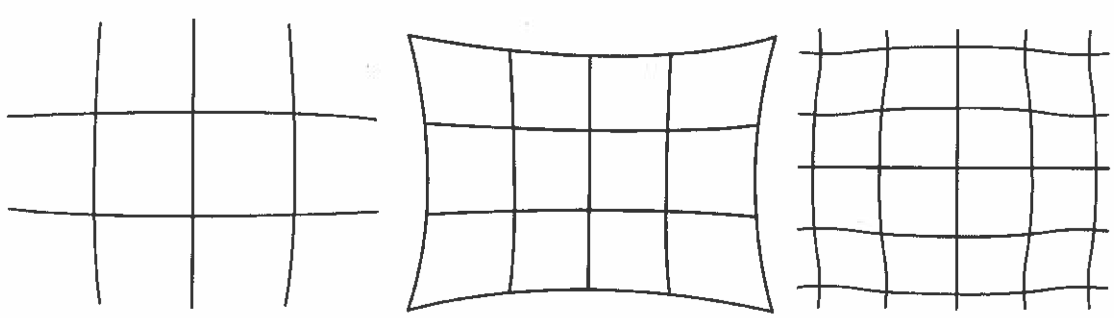
Beams at different portion of the ~~beam~~ lens converge at different point



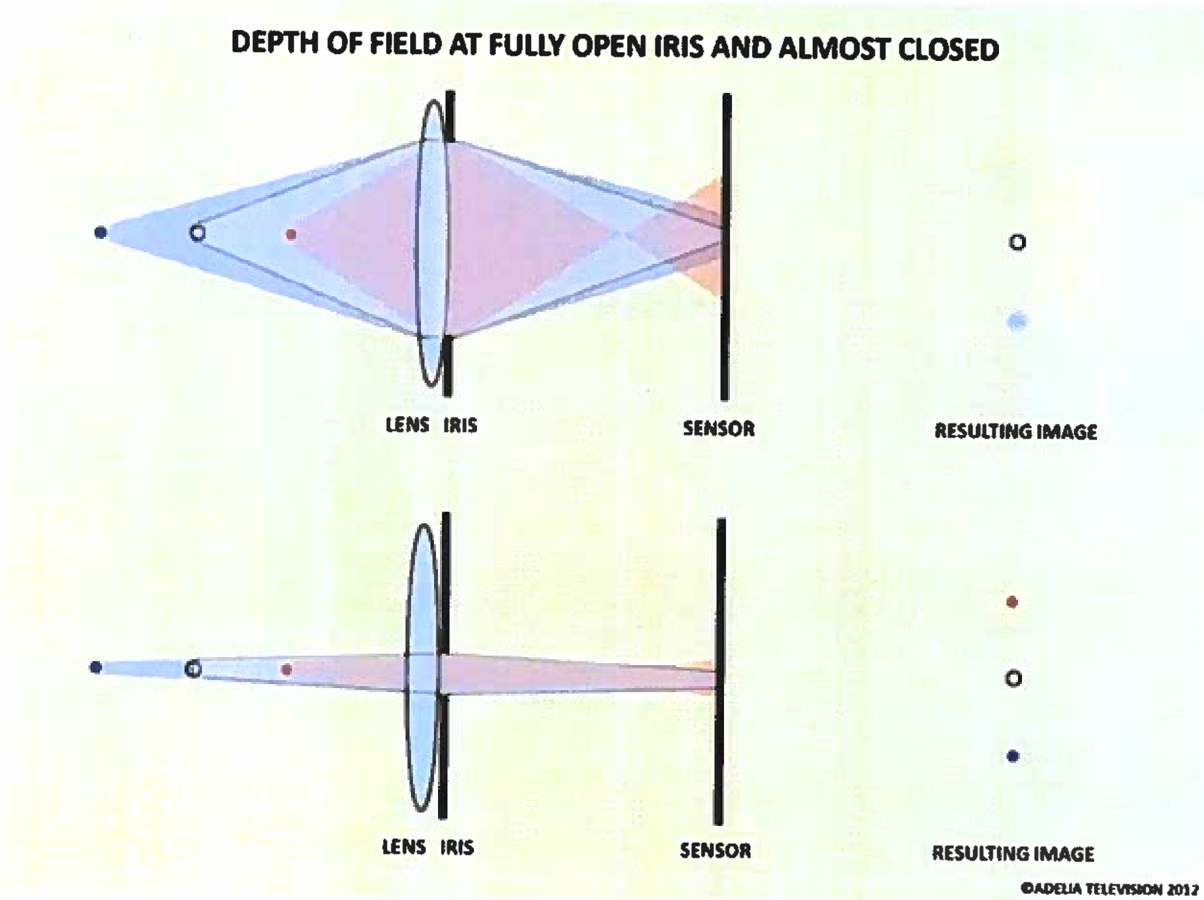
Coma (comatic aberration)

Light entering at an angle is focused at different point and not focused properly

Astigmatism / field curvature



To minimize all monochromatic aberrations, the ~~in~~ light beam size must be smaller than the lens diameter


DEPTH OF FIELD AT FULLY OPEN IRIS AND ALMOST CLOSED
LENS
IRIS
SENSOR
RESULTING IMAGE
LENS
IRIS
SENSOR
RESULTING IMAGE
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