More complex circuits

"Simple" circuits : one battery + multiple resistors

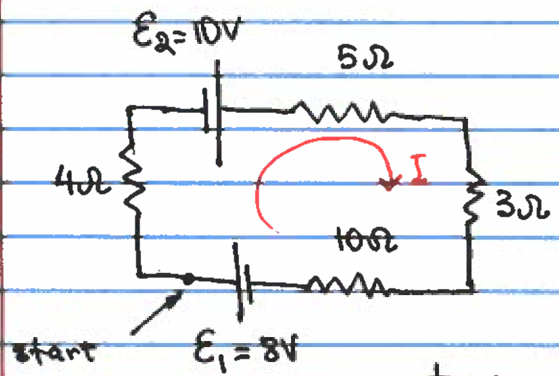
Approach: 1. calculate equivalent resistance R_{eq}

2. Find the total current through the battery $I_{battery} = \mathcal{E}/R_{eq}$

3. If necessary, step back through the circuit to find currents and voltages across individual components

"Complex" circuits: multiple batteryies

Step 1: multiple batteries, one loop of current



All elements are in series, single current flows through everything.

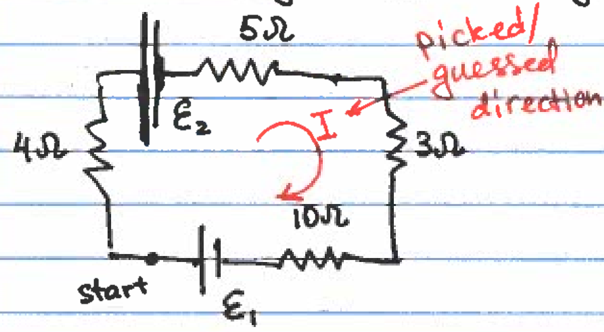
In a single loop we can trace the voltage change across all elements, and check that "round trip" change is zero

$$-I\cdot 4\Omega + \mathcal{E}_2 - I\cdot 5\Omega - I\cdot 3\Omega - I\cdot 10\Omega + \mathcal{E}_2 = 0$$

$$\mathcal{E}_1 + \mathcal{E}_2 - I(22\Omega) = 0$$

$$18\,V = 22\Omega\cdot I \qquad I = 0.82\,A$$

Here I was careful to plug batteries such that they work together.



Here it may not be clear in what direction the current flows. So we pick one, and if it is wrong we'll get a negative value

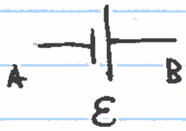
$$-4\Omega\cdot I - \mathcal{E}_2 - 5\Omega I - 3\Omega\cdot I - 10\Omega I + \mathcal{E}_1 = 0$$

$$\underbrace{\mathcal{E}_1 - \mathcal{E}_2}_{-2V} - 22\Omega\cdot I = 0$$

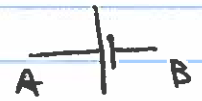
$I = -0.09A$ ← negative value thus our pick of the direction was wrong.

actual current direction

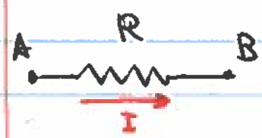
Circuit calculation cheat sheet



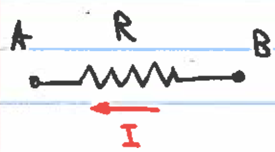
$\Delta V = V_B - V_A = \mathcal{E}$



$\Delta V = V_B - V_A = -\mathcal{E}$

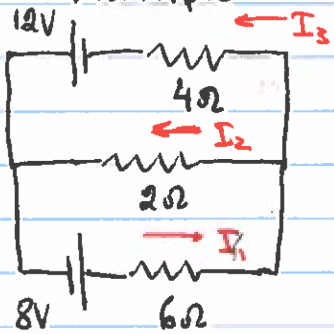


$\Delta V = V_B - V_A = -IR$



$\Delta V = V_B - V_A = IR$

Most complex case: multiple batteries, multiple loops



① Three different currents through each branch. Directions can be randomly chosen, if wrong - the answer comes up with a wrong size

Kirchhoff's rules

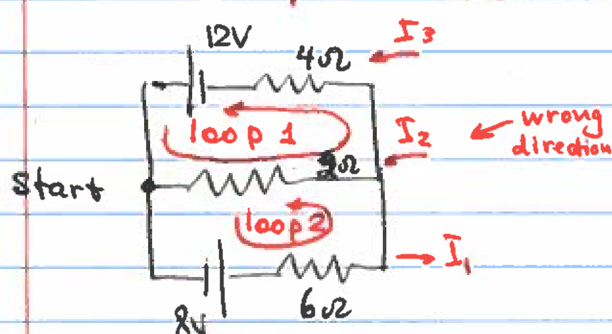
Rule #1: At any junction, the sum of currents flowing in is equal to the sum of currents flowing out $\Sigma I_{in} = \Sigma I_{out}$

$I_1 = I_2 + I_3$

Rule #2: Sum of all potential differences across all elements around any closed loop must be zero

Number of loops needed: # of currents − # of current eqns.

Our example: two independent loops



loop 1: $I_2 \cdot 2\Omega - I_3 \cdot 4\Omega + 12V = 0$ ②

loop 2: $8V - I_1 \cdot 6\Omega + I_2 \cdot 2\Omega = 0$ ③

$I_1 = I_2 + I_3$ ①

① → ③ $8V - (I_2 + I_3) 6\Omega + I_2 \cdot 2\Omega = 0$

$8V - 8\Omega I_2 - 6\Omega I_3 = 0$

② $I_2 \cdot 2\Omega = I_3 \cdot 4\Omega - 12V$

③

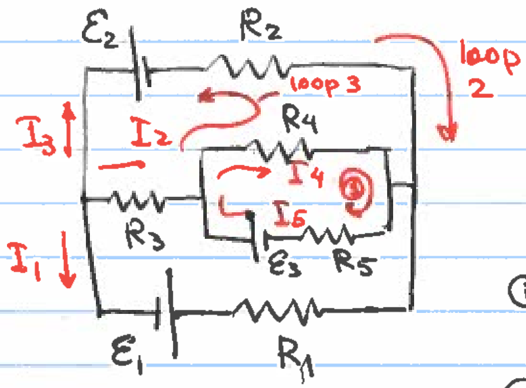
$8V - I_3 16\Omega + 48V - 6\Omega \cdot I_3 = 0$

$56 V = I_3 \cdot 22\Omega$

$I_3 = 28/11$ A

$I_2 = 2I_3 - 6 = -\frac{10}{11}$ A

$I_1 = \frac{28}{11} - \frac{10}{11} = \frac{18}{11}$ A



$$\left.\begin{aligned} I_1+I_2+I_3&=0 \\ I_2&=I_4+I_5 \end{aligned}\right\}\ \text{current rules}$$

5 unknown currents

2 current eqs

need 3 loops

① $-I_4R_4+I_5R_5+\mathcal{E}_3=0$

② $-\mathcal{E}_2-I_3R_2+I_1R_1-\mathcal{E}_1=0$

③ $-I_2R_3-I_4R_4+I_3R_2+\mathcal{E}_2=0$

Thatis where you really want to use computer to solve.