Microscopic definition of entropy

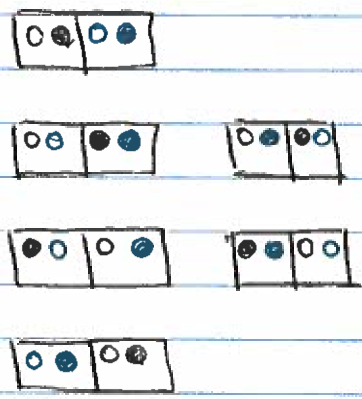
We mentioned that entropy characterizes <u>disorder</u> in a system.

Each macrosopic state can be realized with a number of microsopic configurations

Example : 4 beads in 2 boxes

#1 #2

 – 1 configuration with #2 empty



6 configurations with equal numbers in both sides

 – 1 configurations with #1 empty

W – statistical weight (not work, just the same letter)

$W_{4-0} = 1$ $\quad W_{2-2} = 6$

$W_{0-4} = 1$

Entropy $\underline{S = k_B \ln W}$ Boltzmann definition

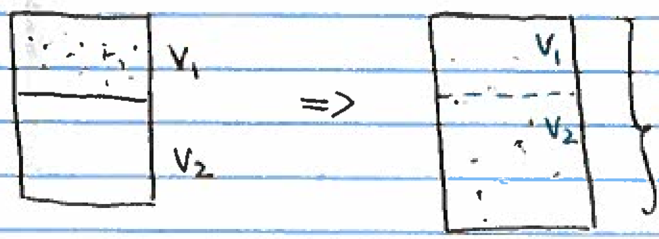
So $S_{4-0} = 0$ and $S_{0-4} = 0$ but $S_{2-2} = k_B \ln 6$

The quote I really like: Boltzman showed that the law of physics are mostly accurate.

Arrow of time appears not because of fundamental laws of physics, but because every complex system evolves from ~~an~~, more ordered (less probable) to less ordered (more probable) state $\longrightarrow$ entropy grows

We calculated the entropy growth when gas expands using macroscopic description. We can do the same with microscopic description as well.

$V_{ini} = V_1$



$V_{final} = V_1 + V_2$

~~Probability of a single molecule~~

For a given volume V_i let's break it into small "cells" of volum V_{min}, and count how many possible "locations" a single molecule can have

$$W_1 = \frac{V_i}{V_{min}}$$

If we have N molecule, each one has the same # of options, so

$$W_i = \left(\frac{V_i}{V_{min}}\right)^N \qquad S_i = k_B \ln W_i = k_B \cdot N \ln \frac{V_i}{V_{min}}$$

If volume is increased $\longrightarrow$ $w_1 = \dfrac{V_{fin}}{V_{min}}$

$$W_{fin} = \left(\frac{V_{fin}}{V_{min}} \right)^N \qquad S_{fin} = k_B \ln W_{fin} = k_B N \ln \frac{V_f}{V_{min}}$$

$$\Delta S = S_{fin} - S_{ini} = k_B \cdot N \left(\ln \frac{V_f}{V_{min}} - \ln \frac{V_{ini}}{V_{min}} \right) = k_B N \cdot \ln \frac{V_f}{V_i}$$

same as $\Delta S = nR \ln V_f/V_i$ we got last time