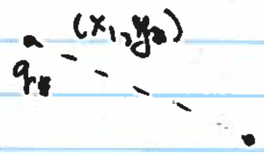
2D electric fields



$\vec{E}(x_2, y_2) = ?$

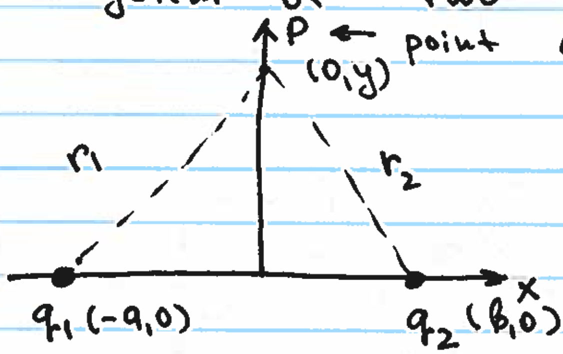
$$\vec{r} = [(x_2 - x_1), (y_2 - y_1)]$$

$$|\vec{r}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$\hat{r} = \left[\frac{x_2 - x_1}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}}, \frac{y_2 - y_1}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}}\right]$$

$$\vec{E} = \frac{kq}{r^2}\hat{r} = \left[\frac{kq\,(x_2-x_1)}{[(x_2-x_1)^2 + (y_2-y_1)^2]^{3/2}}; \frac{kq\,(y_2-y_1)}{[(x_2-x_1)^2 + (y_2-y_1)^2]^{3/2}}\right]$$

System of two charges



$$r_1 = \sqrt{a^2 + y^2}$$

$$r_2 = \sqrt{b^2 + y^2}$$

$$\hat{r}_1 = \left\{\frac{+a}{\sqrt{a^2+y^2}}, \frac{y}{\sqrt{a^2+y^2}}\right\}$$

$$\hat{r}_2 = \left\{\frac{-b}{\sqrt{b^2+y^2}}, \frac{y}{\sqrt{b^2+y^2}}\right\}$$

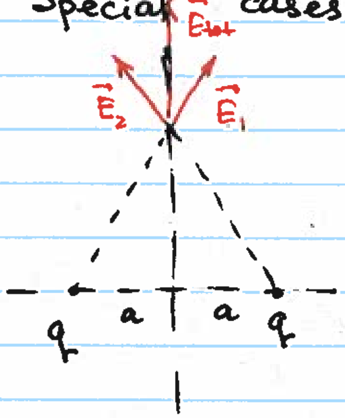
$$\vec{E}_1 = \left[\frac{kq_1 a}{\sqrt{(a^2+y^2)}^{3/2}}; \frac{kq_1 y}{\sqrt{(a^2+y^2)}^{3/2}}\right]$$

$$\vec{E}_2 = \left[\frac{-kq_2 b}{\sqrt{(b^2+y^2)}^{3/2}}; \frac{kq_2 y}{\sqrt{(b^2+y^2)}^{3/2}}\right]$$

$$\vec{E}_{tot} = [E_{1x} + E_{2x}, E_{1y} + E_{2y}] =$$

$$= \left[\frac{kq_1 a}{r_1^3} - \frac{kq_2 b}{r_2^3}; \frac{kq_1 y}{r_1^3} + \frac{kq_2 y}{r_2^3}\right]$$

Special cases $\quad a = b \quad q_1 = q_2 > 0 = q$



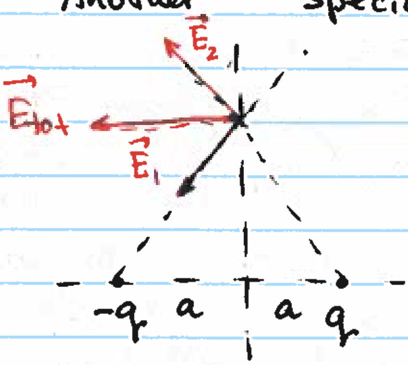
From the symmetry of the problem and/or graphic vector additon we expect $\vec{E}_{tot} = [0, \vec{E}_y]$

indeed since now $r_1 = r_2 = \sqrt{a^2+y^2}$

$$E_{tot\,x} = \frac{kqa - kqa}{b\, r^3} = 0$$

$$E_{tot\,y} = \frac{2kqy}{r^3} = \frac{2kqy}{(a^2+y^2)^{3/2}}$$

Another special case $\quad a = b \quad q_1 = -q_2$



In this case $\vec{E}_{tot} = [E_x, 0]$

$$E_{tot\,x} = -\frac{2kqa}{r^3} = -\frac{2kqa}{(a^2+y^2)^{3/2}}$$

$$E_{tot\,y} = 0$$

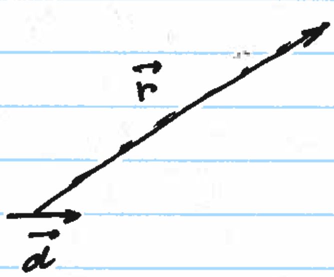
electric dipole

dipole moment $\vec{p} = q \cdot \vec{d}$ $\qquad$ $\vec{d}$ ~~is~~ goes from negative to positive charge

in our case $\vec{d} = [2a, 0]$

$\vec{p} = [2qa, 0]$

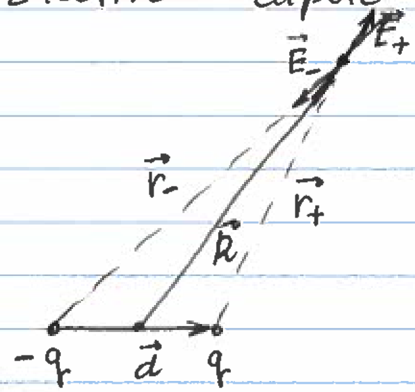
So $\vec{E}_{dipole} = -\dfrac{k\vec{p}}{(a+y)^3}$ $\quad$ if $y >> a$ $\quad \vec{E}_{dipole} \approx -\dfrac{k\vec{p}}{y^3}$



In general

$$\vec{E}_{dipole} = -\frac{k\vec{p}}{r^3}$$

Electric dipole – most common charge configuration



$$\vec{r}_- = \vec{R} + \frac{\vec{d}}{2} \qquad \vec{r}_+ = \vec{R} - \frac{\vec{d}}{2}$$

$$|\vec{r}_\pm| = \sqrt{(\vec{R} + \tfrac{\vec{d}}{2})(\vec{R} + \tfrac{\vec{d}}{2})} =$$

$$= \sqrt{R^2 + \vec{R}\cdot\vec{d} + d^2/4}$$

We can simplify the calculation if we assume that the observation point is far from the dipole $d \ll R$

Then $R^2 \gg \vec{R}\cdot\vec{d} \gg d^2$

$$\sqrt{R^2 + \vec{R}\cdot\vec{d} + d^2} \approx \sqrt{R^2 + \vec{R}\cdot\vec{d}} = R\sqrt{1 + \underbrace{\frac{\vec{R}\cdot\vec{d}}{R^2}}_{\text{small}}}$$

Your favourite Taylor expansion

$$(1+x)^\alpha \approx 1 + \alpha x$$

$$\vec{E}_- = \frac{k(-q)}{r_-^3}\vec{r}_- \qquad \vec{E}_+ = \frac{kq}{r_+^3}\vec{r}_+$$

$$\frac{1}{r_-^3} = \frac{1}{R^3}\left(1 + \frac{\vec{R}\cdot\vec{d}}{R^2}\right)^{-3/2} \approx \frac{1}{R^3}\left(1 - \frac{3}{2}\frac{\vec{R}\cdot\vec{d}}{R^2}\right)$$

$$\frac{1}{r_+^3} \approx \frac{1}{R^3}\left(1 + \frac{3}{2}\frac{\vec{R}\cdot\vec{d}}{R^2}\right)$$

$$\vec{E}_{tot} = \vec{E}_- + \vec{E}_+ = \left(\frac{1}{R^3} - \frac{3}{2}\frac{\vec{R}\cdot\vec{d}}{R^5}\right)\left(\vec{R} + \frac{\vec{d}}{2}\right)(-kq) +$$

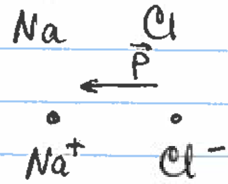
$$+ \left(\frac{1}{R^3} + \frac{3}{2}\frac{\vec{R}\cdot\vec{d}}{R^5}\right)\left(\vec{R} - \frac{\vec{d}}{2}\right)(kq) =$$

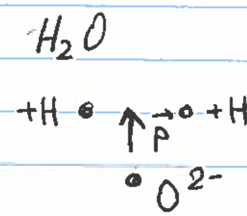
1 2 3 1 2 3

1 and 3 cancel out!

$$\vec{E}_{tot} = -\frac{kq\vec{d}}{R^3} = -\frac{k\vec{p}}{R^3} \qquad \vec{p} = q\cdot\vec{d} \text{ dipole moment}$$

Polar molecules





Special unit for a molecular dipole moment
1 Debay $= 10^{-20}$ C·m

Does an electron has a dipole moment?
We consider electron to pe a point-like particle $\rightarrow$ no internal ~~stru~~ charge structure.
Many physicists are trying to measure EDM $\rightarrow$ we know that it is

Dipole's electrostatic potential

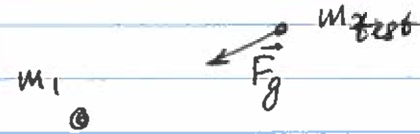
$$V = V_+ + V_- = \frac{kq}{r_+} - \frac{kq}{r_-} =$$

$$= \frac{kq}{R\sqrt{1+\frac{\vec R \vec d}{R^2}}} - \frac{kq}{R\sqrt{1+\frac{\vec R \vec d}{R^2}}} \approx \frac{kq}{R}\left(1+\frac{\vec R \vec d}{2R^2}\right) - \frac{kq}{R}\left(1-\frac{\vec R\cdot\vec d}{2R^2}\right)$$

$$= \frac{kq\cdot \vec R\cdot\vec d}{R^3} = \frac{k\vec p\cdot\vec R}{R^3}$$

Electric potential

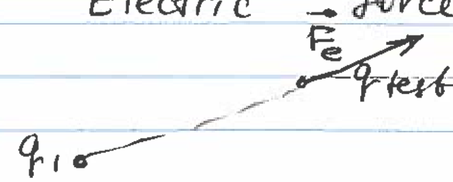
Gravitational force



$$\vec{F}_g = -\frac{G m_1 m_{test}}{r^2}\hat{r}$$

$$\vec{g} = \vec{F}_g / m_{test} \quad \left(\text{acceleration due to gravity}\right)$$

Electric force



$$\vec{F}_e = \frac{k q_1 q_{test}}{r^2}\hat{r}$$

electric field $\quad \vec{F}_e / q_{test}$

$$\vec{E} = \frac{k q_1}{r^2}\hat{r}$$

Gravitational potential energy

$$U_g = -\frac{G m_1 m_{test}}{r}$$

Electrostatic potential energy

$$U_e = \frac{k q_1 q_{test}}{r}$$

Electric potential

(often φ) $\quad V = \dfrac{U_e}{q_{test}} = \dfrac{k q_1}{r}$

Connection b/w electric potential and electric field

$$\vec{E} = \left(-\frac{\partial V}{\partial x}\right)\hat{i} + \left(-\frac{\partial V}{\partial y}\right)\hat{j} + \left(-\frac{\partial V}{\partial z}\right)\hat{k} = -\nabla V$$

Example: $\quad V = \dfrac{k q_1}{r} = \dfrac{k q_1}{\sqrt{x^2+y^2}}$

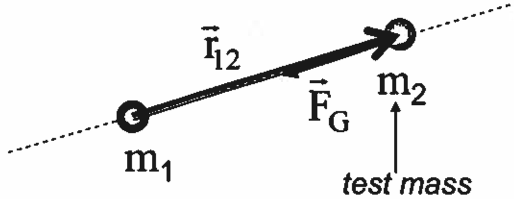
$$\frac{\partial V}{\partial x} = k q_1 \frac{\partial \; 1/\sqrt{x^2+y^2}}{\partial x} = k q_1 \left(-\frac{1}{\cancel{2}}\right)\frac{\cancel{2}x}{(x^2+y^2)^{3/2}} = -\frac{k q_1 x}{(x^2+y^2)^{3/2}}$$

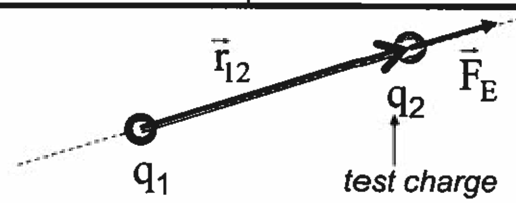
$$E_x = -\frac{\partial V}{\partial x} = \frac{k q_1 x}{(x^2+y^2)^{3/2}} \qquad E_y = -\frac{\partial V}{\partial y} = \frac{k q_1 y}{(x^2+y^2)^{3/2}}$$

Gravity vs Electricity

Gravitational force	$\vec{F}_G = -G\dfrac{m_1 m_2}{r^2}\hat{r}$	Electrostatic force	$\vec{F}_E = k\dfrac{q_1 q_2}{r^2}\hat{r}$
Acceleration due to gravity	$\vec{g} = -G\dfrac{m_1}{r^2}\hat{r}$	Electric field	$\vec{E} = k\dfrac{q_1}{r^2}\hat{r}$
Gravitational potential energy	$U_G = -G\dfrac{m_1 m_2}{r}$	Electrostatic potential energy	$U_E = k\dfrac{q_1 q_2}{r}$
		Electric potential	$V_E = \dfrac{U_E}{q_2} = k\dfrac{q_1}{r}$

φ





$V_{AB} = \varphi_B - \varphi_A$