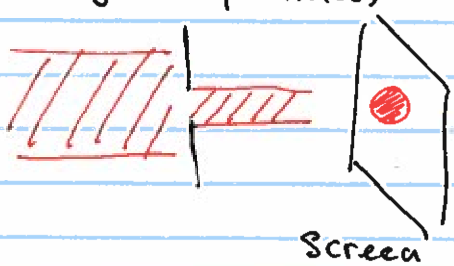
Diffraction

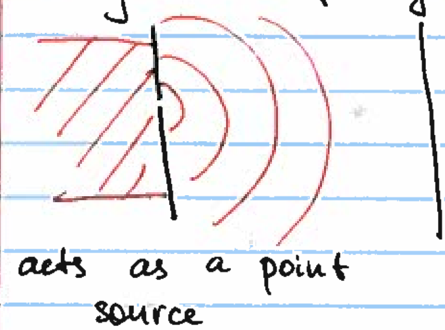
Diffraction is also an interference phenomena. We usually reserve this term to describe what happens to light when it passes through apertures or edges.

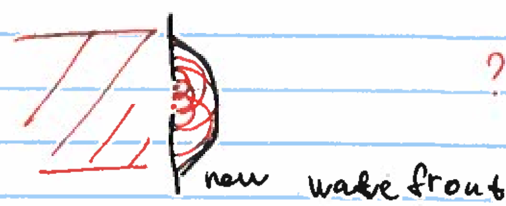
Naive expectation
(or light as particles)



a spot matching a size of the opening

Huygens principle
Very small opening



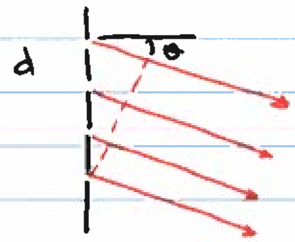


How to calculate the diffraction image?

Method 1: imaging ~~the~~ the opening as a combination of many slits

Method 2: exact calculations (with complex waves)

Reminder: Interference pattern from N slits at distance d from each other



Each consequitive slit introduces delay of $d\sin\theta$ or phase delay $2\pi d\sin\theta/\lambda$

$$I_{N\,slits} = I_1 \left(\frac{\sin\left(\frac{\pi N d\sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi d\sin\theta}{\lambda}\right)} \right)^2$$

I_1 – intensity from one slit

Method 1: ~~[illegible]~~ an opening of size $D = dN$

$\Rightarrow$ $d \to 0$ and $N \to \infty$, $I_1 = I_0/N^2$

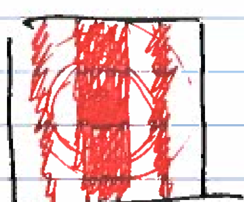
↑ total intensity

$$\sin\frac{\pi N d\sin\theta}{\lambda} \Rightarrow \sin\frac{\pi D\sin\theta}{\lambda}$$

$$\sin\frac{\pi d\sin\theta}{\lambda} \approx \frac{\pi d\sin\theta}{\lambda}$$

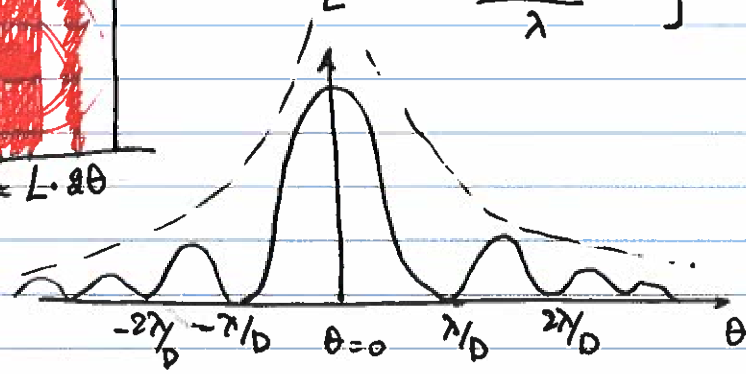
$$I_{opening} = \frac{I_0}{N^2}\left[\frac{\sin\left(\frac{\pi D\sin\theta}{\lambda}\right)}{\frac{\pi d\sin\theta}{\lambda}}\right]^2 =$$

$$= I_0 \left[\frac{\sin\frac{\pi D\sin\theta}{\lambda}}{\frac{\pi D\sin\theta}{\lambda}}\right]$$



spot size $\approx L\cdot 2\theta$

$= 2L\cdot\frac{\lambda}{D}$

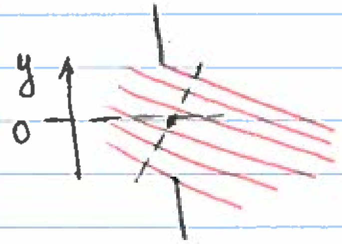


First diffraction minimum

$$\sin\frac{\pi D\sin\theta_{min}}{\lambda} = 0$$

$$\frac{D\sin\theta_{min}}{\lambda} = 1$$

$$\theta_{min} \approx \sin\theta_{min} = \frac{\lambda}{D}$$ for a 1D opening



For any height above the center of the slit y, the retardation (compare to the center) is $y \cdot \sin\theta$

$-a/2 < y < a$ — slit.

$$k = \frac{2\pi}{\lambda}$$

The electric field in each point along the wavefront $dE(y) = E_0 \frac{dy}{a} e^{ik \cdot y \cdot \sin\theta}$

$$\left\{ \int e^{\alpha x} dx = e^{\alpha x}/\alpha \right.$$

Total electric field

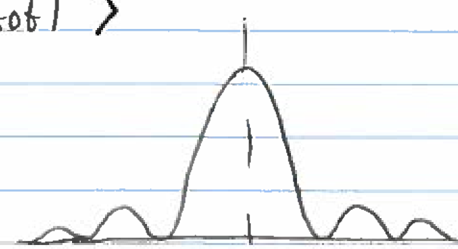
$$E_{tot} = \int_{-a/2}^{a/2} dE_y = \frac{E_0}{a} \int_{-a/2}^{a/2} e^{ik\sin\theta \cdot y} dy =$$

$$= \frac{E_0}{a} \frac{1}{ik\sin\theta} \underbrace{\left[e^{ik\sin\theta\, a/2} - e^{-ik\sin\theta\, a/2} \right]}_{2i \sin\left[k\sin\theta a/2\right]} =$$

$$= \frac{E_0}{a} \frac{2}{k} \frac{\sin\left[k\sin\theta \cdot a/2\right]}{\sin\theta} = E_0 \frac{\sin\left[\frac{ka}{2}\sin\theta\right]}{\left[\frac{ka}{2}\sin\theta\right]}$$

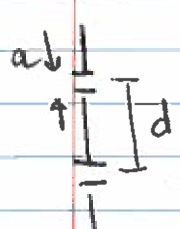
Power (or intensity) $P \propto \langle |E_{tot}|^2 \rangle$

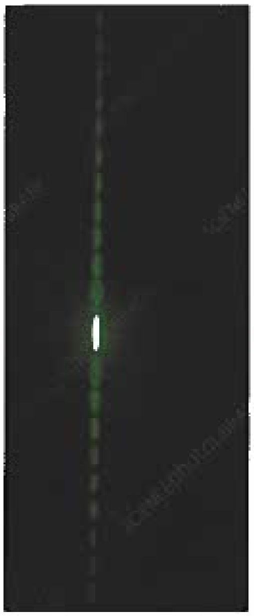
$$P = P_0 \frac{\sin^2\left[\frac{\pi a}{\lambda}\sin\theta\right]}{\left(\frac{\pi a}{\lambda}\sin\theta\right)^2}$$

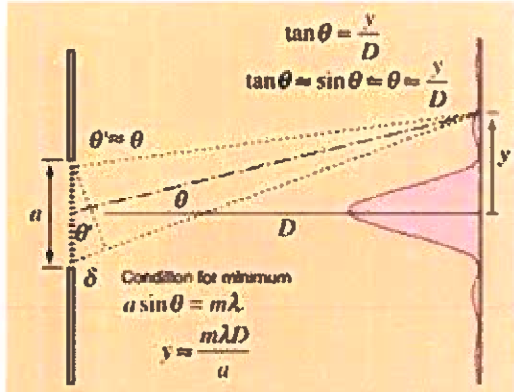


Two slits next to each other

$$P = P_0 \underbrace{\frac{\sin^2\left[\frac{\pi a}{\lambda}\sin\theta\right]}{\left(\frac{\pi a}{\lambda}\sin\theta\right)^2}}_{\text{single-slit diffraction}} \cdot \underbrace{\cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)}_{\text{two-slit interference}}$$







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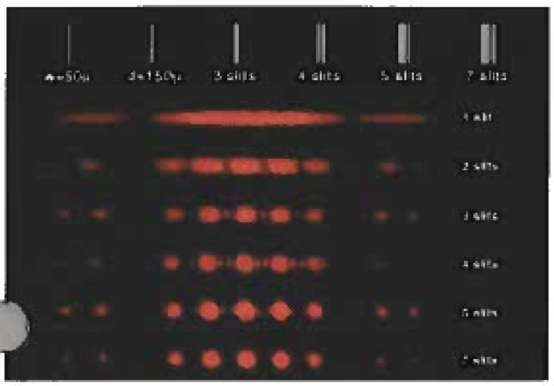
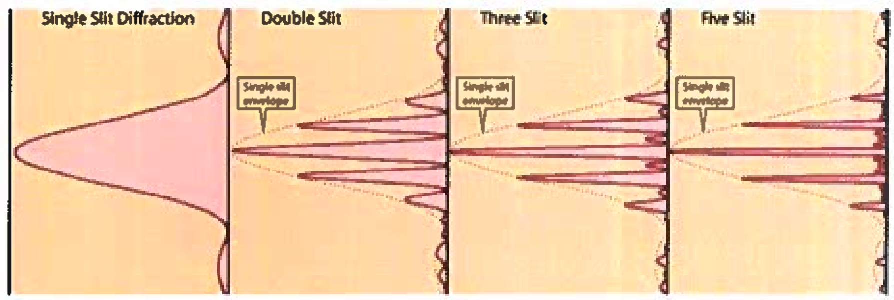
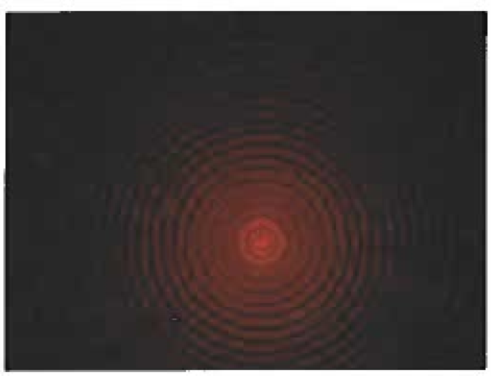
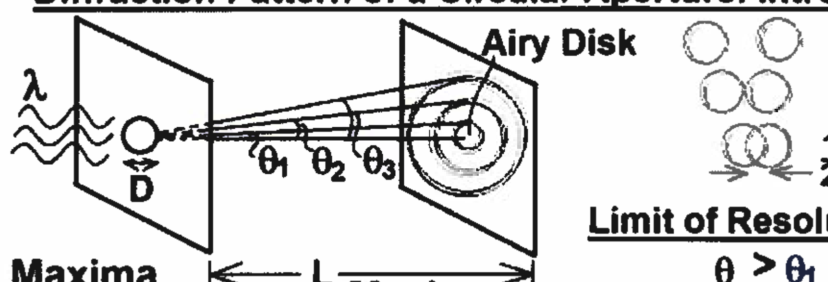






Diffraction Pattern of a Circular Aperture: Intro



Limit of Resolution

$\theta \geq \theta_1$

Maxima

$\sin\theta_1 = 1.22\frac{\lambda}{D}$

$\sin\theta_2 = 2.23\frac{\lambda}{D}$

$\sin\theta_3 = 3.24\frac{\lambda}{D}$

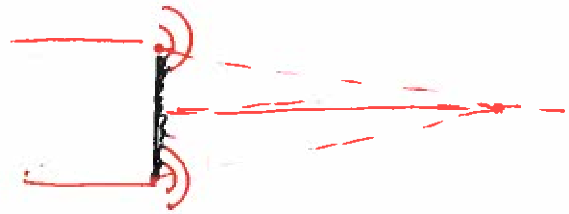
Maxima

$\sin\theta_{Max\,1} = 1.63\frac{\lambda}{D}$

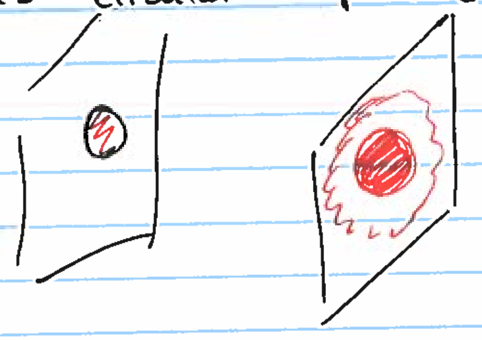
$\sin\theta_{Max\,2} = 2.68\frac{\lambda}{D}$

$\sin\theta_{Max\,3} = 3.70\frac{\lambda}{D}$

Arago spot - bright spot in the center of a circular target "shadow"



2D circular aperture



$$I(\theta) = I_0 \left(\frac{J_1\left(\frac{\pi D \sin\theta}{\lambda}\right)}{\left(\frac{\pi D \sin\theta}{\lambda}\right)} \right)^2$$

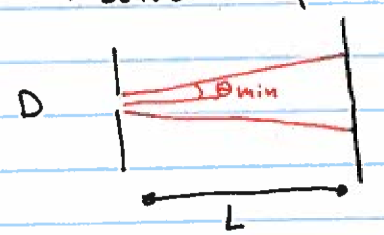
$J_1(x)$ - Bessel function of the first kind

$J_1(x) = 0$ for $x = 3.83$

$$\frac{\pi D \sin\theta_{min}}{\lambda} = 3.83$$

$$\boxed{\theta_{min} \approx 1.22 \frac{\lambda}{D}}$$

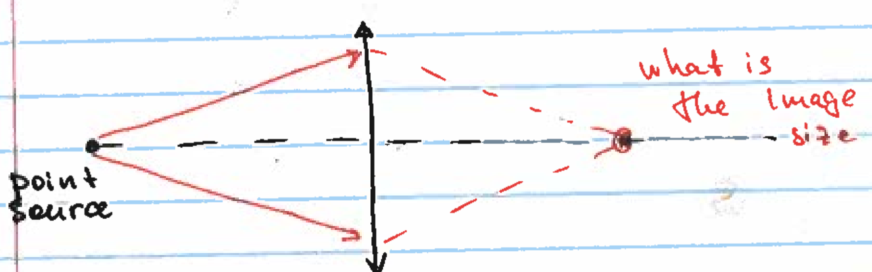
Optical resolution - how well one can resolve optical features



beam spot size $\approx L\theta_{min}$

The smaller D is, the larger is θ_{min} → larger spot on the screen.

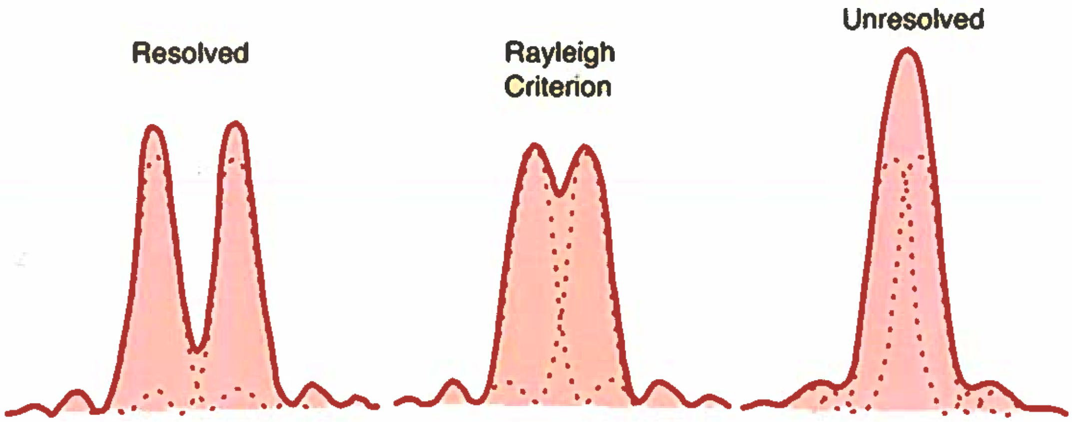
Same equation will ~~*~~ be valid for a lens



We cannot recreate the image to be a point source exactly, since we cannot collect all the light emitted by the source!

$$\boxed{\text{min spot} \propto f \cdot \frac{\lambda}{D}}$$

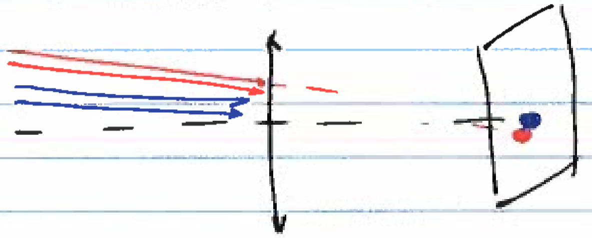
larger lens → better sharpness



$\sin\theta_R = \dfrac{\lambda}{d}$	$\sin\theta_R = 1.22\dfrac{\lambda}{d}$
Single slit	**Circular aperture**

Why the telescopes have to be large?

To resolve better!



Rayleigh criteria: Two objects can be resolved if their angular separation is larger than θ_{min}

How well our eyes can do?

Assuming a ~~pixel~~ ~~size~~ pupil size $D \approx 5mm$ and average visible light $\lambda \approx 500nm = 5\cdot10^{-7}m$

$$\theta_{min} = 1.22\frac{\lambda}{D} = 1.22\cdot\frac{5\cdot10^{-7}m}{5\cdot10^{-3}m} \approx 1.22\cdot10^{-4}\ rad$$

~~angular minutes~~

$$= 180^\circ/rad\cdot 60\frac{min}{deg}\cdot 1.22\cdot10^{-4}\ rad \approx 1' \text{ (angular minute)}$$

How far you can be to resolve $\Delta x = 1$-mm size feature

$$L_{min} = \frac{\Delta x}{\theta_{min}} = \frac{10^{-3}}{1.22\cdot10^{-4}} \approx 8m$$

(if they are bright enough!)

How to increase the resolution?

- Use shorter wavelength
  moving from CD (808nm) to DVD (405nm) allowed to halve the size of features and quadruple the information capacity
- X-ray diffraction (XRD) – $\lambda_{x\text{-ray}} = 0.03 - 3nm$
- Modern chip technology uses 10nm wavelength (extreme UV light)

In 2030 they plan to reach a few angstrom (10^{-10} m)

- To resolve even smaller features → use heavier particles

Anything can display wave properties

$\lambda = \frac{\hbar}{p}$ De-Broigle wavelength

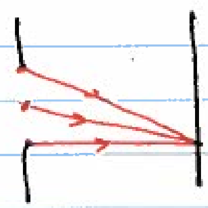
$\hbar = 10^{-34}$ J·s Plank's constant

p – particle momentum

Electron diffraction is a common tool for material science

Neutrons are also used for material science to probe deeper into material (since they don't have electric charge)

- Near-field imaging – collecting and analyzing light at distances close to the ~~fea~~feature size



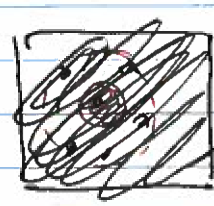
Near-field images contain much more information and can provide much better resolution → but hard to collect

(Check out Dr. Mumtaz Qazilbash research)

- Nonlinear sub-~~[illegible]~~ diffraction imaging

normal laser beam

Large, diffraction-limited laser spot excites many objects, making it impossible to resolve the individual contributions



~~structured~~ only unsaturated source is visible

"blinding" light

STED imaging (nobel prize 2016)

Vortex (donut) beam "blindes" all objects except for one sitting in the center, where its intensity is zero