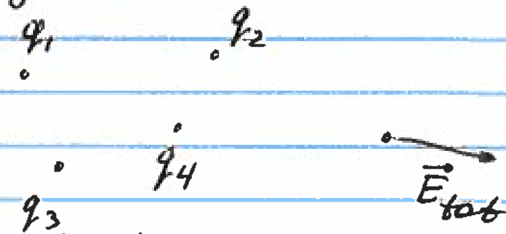
Continuous charge distribution

Many point charges

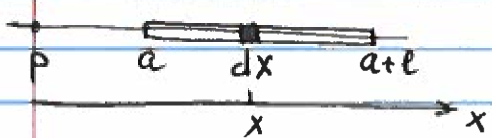


$$\vec{E}_{tot} = \sum_{i=1}^{N} \frac{kq_i}{r_i^2}\hat{r}_i$$

Continuously charged object $\longrightarrow$ break its volume into small point charges

summation $\longrightarrow$ integration

1D world



Total charge $- Q$

$dq = Q\, dx/\ell$

$$dE = \frac{kdq}{x^2} = \frac{kQ}{\ell}\frac{dx}{x^2}$$

$$V = \int_a^{a+\ell} \frac{kQdx}{\ell x} = $$

$$= \frac{kQ}{\ell} \ln\Big|_a^{a+\ell} = \frac{kq}{\ell}\ln\left(\frac{a+\ell}{a}\right)$$

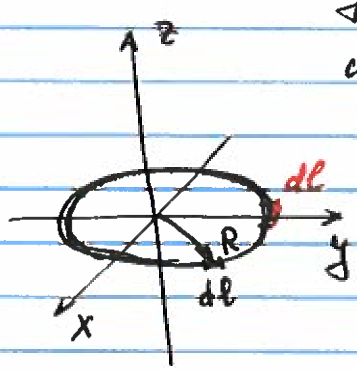
$$E = \int_a^{a+\ell} dE = \frac{kQ}{\ell}\int_a^{a+\ell}\frac{dx}{x^2} \rightarrow$$

$$= -\frac{kQ}{\ell}\frac{1}{x}\Big|_a^{a+\ell} = \frac{kQ}{\ell}\left(\frac{1}{a} - \frac{1}{a+\ell}\right)$$

$$= \frac{kq}{a(a+\ell)}$$

3D or 2D world $\longrightarrow$ let's look for some helpful symmetries.

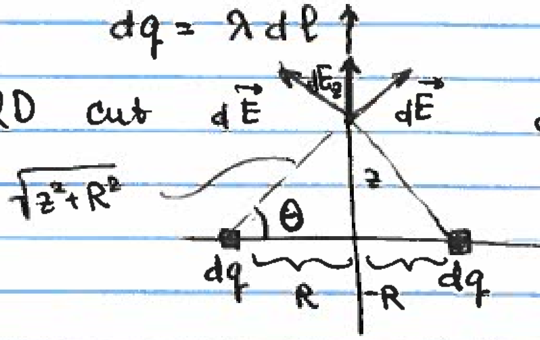
Example 1: Find the electric field on the ~~sy~~ central axis of a uniformly charged ring



Charge density $\lambda = Q/2\pi R$

$dq = \lambda\, dl$

2D cut



$dE_z = dE \sin\theta$

$\sin\theta = \frac{z}{\sqrt{z^2+R^2}}$

only z component of $\vec{E}$-field remains

From a single dq $\quad dE = \frac{k\,dq}{(z^2+R^2)}$

$$dE_z = \frac{k\,dq}{(z^2+R^2)} \sin\theta = \frac{k\,dq \cdot z}{(z^2+R^2)^{3/2}}$$

$$E_z = \int_{\substack{\text{along} \\ \text{the ring}}} \frac{kz\,\lambda\, dl}{(z^2+R^2)^{3/2}} = \frac{kz\lambda}{(z^2+R^2)^{3/2}} \int dl =$$

$$= \frac{kz\lambda}{(z^2+R^2)^{3/2}} \cdot 2\pi R = \frac{kQz}{(z^2+R^2)^{3/2}}$$

$$V = \int \frac{k\,dq}{\sqrt{R^2+z^2}} = \frac{kQ}{\sqrt{R^2+z^2}}$$