Electric circuits with batteries and resistors

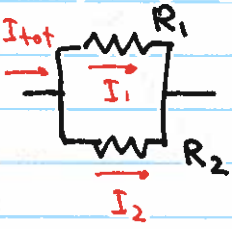
Series connection

$I_1 = I_2 = I$ same current

$V_1 = IR_1 \quad V_2 = IR_2$

$V_{tot} = V_1 + V_2$

Parallel connection



$V_1 = I_1 R_1$

$V_2 = I_2 R_2$

$V_1 = V_2$

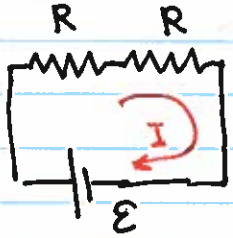
$I_1 + I_2 = I_{total}$

Connection to the battery

$$I_0 = \frac{\varepsilon}{R}$$

$$P_0 = I \cdot V = \frac{\varepsilon^2}{R}$$

Series connection



$$I_s = \frac{\varepsilon}{2R}$$ (lower than for one resistor)

Power dissipated on a single bulb

$$P = I^2 \cdot R = \frac{\varepsilon^2}{4R^2} \cdot R = \frac{\varepsilon^2}{4R}$$

Four time dimmer than a single bulb

Parallel connection

Current through each resistor

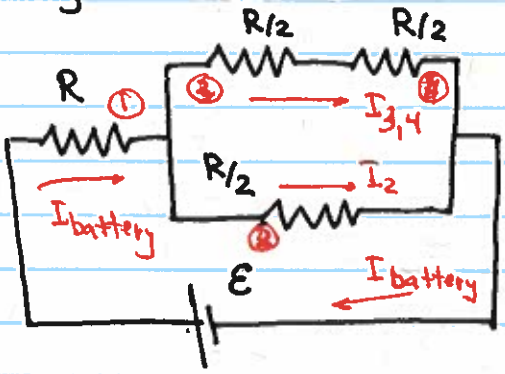
$$I_p = \frac{\varepsilon}{R}$$ same as for a single resistor

Power dissipated on a single resistor

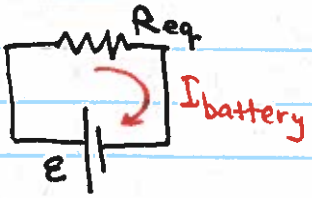
$P = I_p^2 \cdot R = \varepsilon^2/R$ same

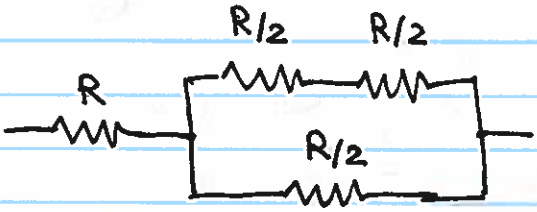
Total current drawn from the battery $I = 2I_p = \frac{2\varepsilon}{R}$

Circuits with one battery and many resistors

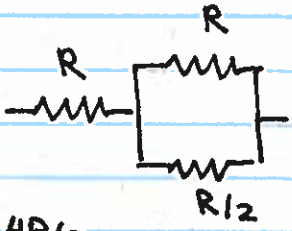


1. Lets find $I_{battery}$ by calculating the equivalent resistance





$\Rightarrow$



$\Rightarrow$

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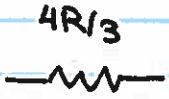


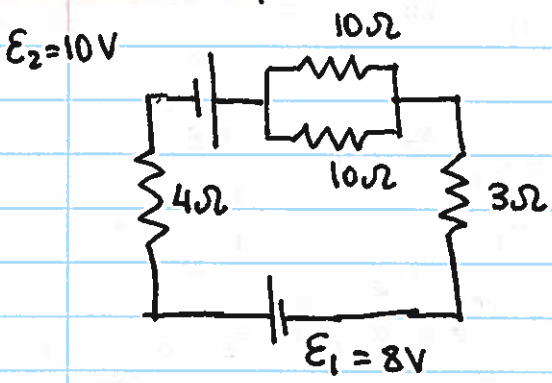
$R_{eq} = 4R/3$

$$I_{battery} = \mathcal{E}/R_{eq} = \frac{3\mathcal{E}}{4R}$$

What is the current and the voltage drop on ~~the~~ each of the resistors?

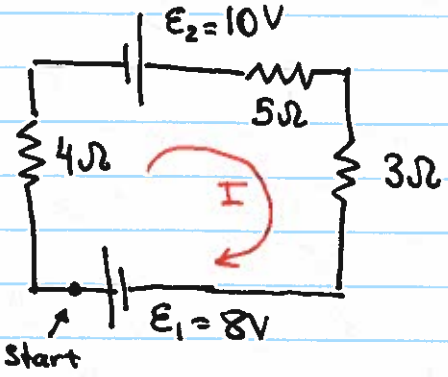
Resistor	Current	Voltage
1	$I_1 = I_{battery}$	$V_1 = R \cdot I_{battery} = 3\mathcal{E}/4$
2	$I_2 = \frac{V_2}{R/2} = \frac{\mathcal{E}}{2R}$	$V_2 = \mathcal{E} - 3\mathcal{E}/4 = \mathcal{E}/4$
3,4	$I_{3,4} = I_{battery} - I_2 = \frac{\mathcal{E}}{4R}$	$V_3 = V_4 = I_{3,4} \cdot \frac{R}{2} = \frac{\mathcal{E}}{8}$

More complex circuits with multiple batteries



~~can be~~ equivalent to a single loop

=>

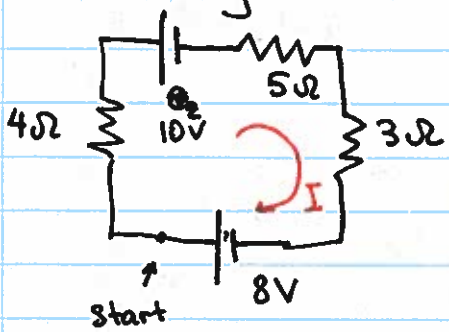


In a single loop total voltage drop must be zero

$$-4\Omega \cdot I + 10V - 5\Omega \cdot I - 3\Omega I + 8V = 0$$

$$18V = 12\Omega \cdot I \qquad I = 3/2\,A$$

What if I accidentally connect the second battery back wards?



$$-4\Omega \cdot I - 10V - 5\Omega \cdot I - 3\Omega \cdot I + 8V = 0$$

$$-2V = 12\Omega \cdot I$$

$$I = -\frac{1}{6}A$$

Minus sign means we guessed the wrong direction for the current