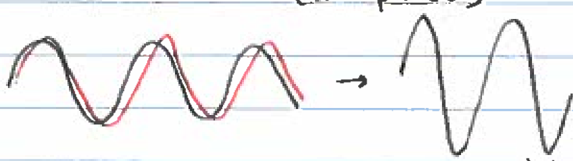
Wave optics

In ray optics we did not consider wave nature of e-m field

Historically: carpuscular nature of light
~~m~~ (light is a particle)
more particles → brighter spot (like in paintball)

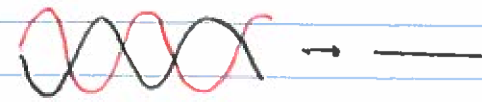
Waves display interference!

Constructive (in-phase)



x2 amplitude

Destructin (out of phase)



no wave!

How do we see an e-m wave

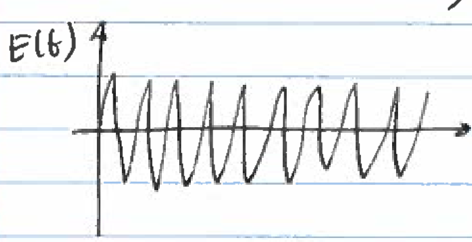
$$E = E_0 \cos(kx - \omega t) = E_0 \cos\left(\frac{2\pi}{\lambda} \cdot x - \frac{2\pi}{T} \cdot t\right)$$

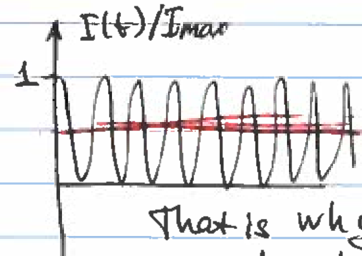
Visible light: $\lambda = 400\text{-}800\,nm \rightarrow 600\,nm$ (yellow-~~green~~)
$T = \lambda / c = 6 \cdot 10^{-7} m / 3 \cdot 10^{8} m/s = 2 \cdot 10^{-15} s = 2 fs$

Much-much faster than any human eye, or even any of electronics detector!

We only see a time-averaged intensity

$$I = \frac{1}{\cancel{2}\mu_0 c} \langle E^2 \rangle = \frac{1}{\cancel{2}\mu_0 c} E_0^2 \left\langle \cos^2\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T}\right)\right\rangle = \frac{1}{2\mu_0 c} E_0^2$$

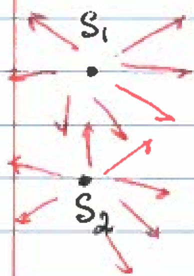




average 1/2

That is why a laser beam appears to be steady and unchanging in time

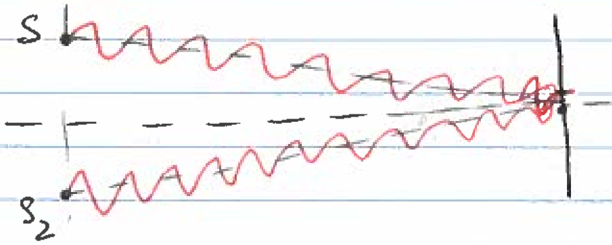
Two-slit interference



$E_1 = E_0 \cos(k\cancel{x_1}r - \cancel{i}\omega t) \rightarrow E_0 e^{ik\cancel{x}r - i\omega t}$

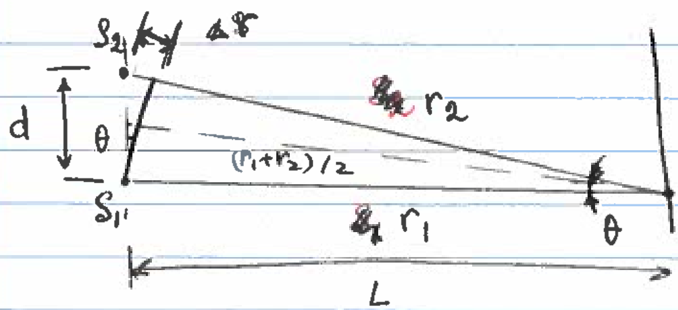
$E_2 = E_0 \cos(kx_2 - \omega t) \rightarrow E_0 e^{ikx_2 - i\omega t}$

Two identical sources in different locations



~~the~~ center is equidistant from the two sources, two arriving waves are identical $\rightarrow$ constructive interference

If we move away from the center, the distance b/w the observation point and ~~the~~ each sorce changes. Distrance difference $\Delta x = \lambda/2 \rightarrow$ destructive interference then when $\Delta x = \lambda \rightarrow$ constructive again



$r_2 - r_1 = \Delta r \approx d \sin\theta$
if $d \ll L$

Interference signal

$E_{total} = E_0 e^{ikr_1 - i\omega t} + E_0 e^{ikr_2 - i\omega t} =$

$= E_0 e^{ik\left(\frac{r_1+r_2}{2}\right) - i\omega t} e^{ik(r_1 - r_2)/2} + E_0 e^{ik\left(\frac{r_1+r_2}{2}\right) - i\omega t} e^{-ik(r_1-r_2)/2}$

$= E_0 e^{ik\left(\frac{r_1+r_2}{2}\right) - i\omega t} \underbrace{\left[e^{ik\Delta r/2} + e^{-ik\Delta r/2} \right]}_{2\cos(k\Delta r/2)} =$

$= \underbrace{2E_0 \cos\left(\frac{k\Delta r}{2}\right)}_{\text{local wave}} \cdot e^{ik\left(\frac{r_1+r_2}{2}\right) - i\omega t}$ amplitude

$E(\theta) = 2E_0 \cos \frac{kd\sin\theta}{2}$

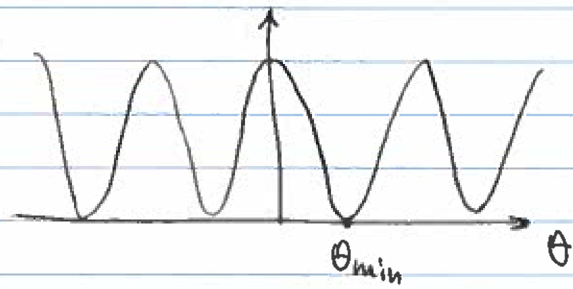
Physical

$$E_{tot} = \mathrm{Re}\left[2E_0 \cos\frac{k\Delta r}{2} \cdot e^{ik\frac{r_1+r_2}{2} - i\omega t}\right] =$$

$$= 2E_0 \cos\frac{k\Delta r}{2} \cos\left(\frac{k(r_1+r_2)}{2} - \omega t\right)$$

$$I = \frac{1}{\mu_0 c}\langle E_{tot}^2 \rangle = 4\frac{E_0^2}{\mu_0 c} \cos^2\frac{k\Delta r}{2} \cdot \frac{1}{2} =$$

$$= 4 I_{one} \cos^2\left(\frac{k}{2} d \sin\theta\right)$$



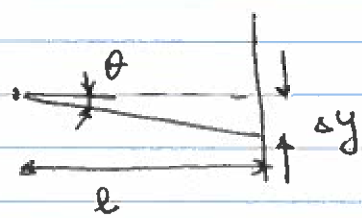
$$\frac{k}{2} d \sin\theta_{min} = \frac{\pi}{2}$$

$$\frac{\cancel{\pi}}{\lambda} d \sin\theta_{min} = \frac{\cancel{\pi}}{2}$$

$$\sin\theta_{min} \approx \theta_{min} = \lambda/2d$$

Interference fringes

What is the distance b/w the stripes on the screen?



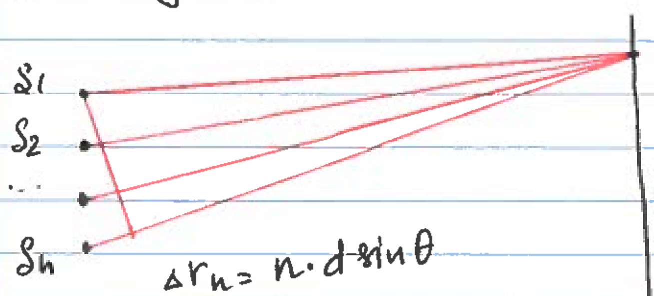
$\Delta y \approx \ell \cdot \tan\theta \approx \ell \cdot \theta$
(make sure θ is in rad!)

$$\Delta y = \ell \cdot \theta$$

$$\Delta y_{min} = \ell \cdot \theta_{min} = \ell\,\lambda/2d$$

Many slits

N total



$\Delta r_n = n \cdot d \sin\theta$

$$E_{tot} = E_0 e^{ikr_1 - i\omega t} + E_0 e^{ikr_1 + ik\Delta r - i\omega t} + E_0 e^{ikr_2 + 2ik\Delta r} + \dots =$$

$$= E_0 e^{ikr_1 - i\omega t}\left[1 + e^{ik\Delta r} + e^{i2k\Delta r} + \dots\right]$$

geometric progression

$$= E_0 e^{ikr_1 - i\omega t} \frac{1 - e^{ikN\Delta r}}{1 - e^{ik\Delta r}}$$

$$\longrightarrow E_0 \cos(kr - i\omega t) \underbrace{\frac{\sin kN\Delta r}{\sin k\Delta r}}_{\to N \text{ for } k\Delta r \to 0}$$

$I_{tot} = N^2 I_0$ at maxima