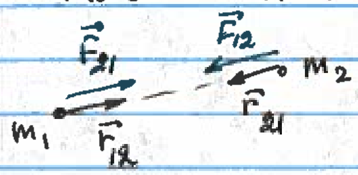
Electric charge and electric force

Gravity

mass $m > 0$



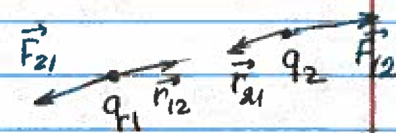
$$\vec{F}_{12} = \frac{G m_1 m_2}{r_{12}^2}\hat{r}_{21} = -\frac{G m_1 m_2}{r_{12}^2}\hat{r}_{12}$$

always attractive

Electricity

charge q (can be positive or negative) or zero

units - Coulumb



$$\vec{F}_{12} = \frac{k q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

if $q_1 q_2 > 0$ — repulsive
$q_1 q_2 < 0$ — attractive

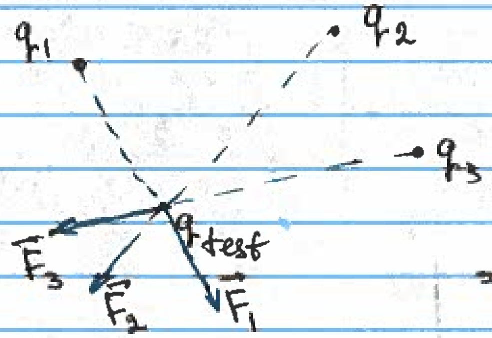
[k - Coulumb constant $\quad k = 9 \cdot 10^{9} \frac{N \cdot m^2}{C^2}$

alternative : $k = \frac{1}{4\pi\varepsilon_0}$ $\quad \varepsilon_0$ - permittivity of vacuum

we need those because of "random" definition of SI units $\quad \varepsilon_0 = 8.85 \cdot 10^{-12} \frac{C^2}{N \cdot m^2}$

(For the same reason 1 Coulumb is a <u>huge</u> charge

Electric forces are vectors!



$$\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 =$$

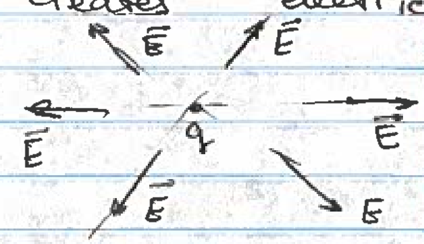
$$= \frac{k q_1 q_t}{r_1^2}\hat{r}_1 + \frac{k q_2 q_t}{r_2^2}\hat{r}_2 + \frac{k q_3 q_t}{r_3^2}\hat{r}_3$$

$$= q_t \underbrace{\left[\frac{k q_1}{r_1^2}\hat{r}_1 + \frac{k q_2}{r_2^2}\hat{r}_2 + \frac{k q_3}{r_3^2}\hat{r}_3\right]}_{\text{electric field } \vec{E}_{total}}$$

Electric field describes the "landscape" due to all surrounding charges, so that
$\vec{F}_{test} = q_{test} \cdot \vec{E}$

A single charge creates electric field

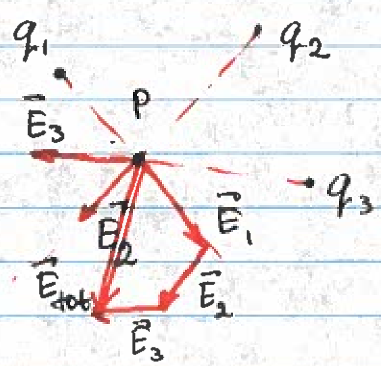
$\vec{E} = \frac{kq}{r^2}\hat{r}$



Electric force lines help visualize E-field direction
At any point $\vec{E}$ is tangential to the field line.

Similarly, the electric fields produced by different sources add up as vectorse

$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots$



That means that

$E_{tot\,x} = E_{1x} + E_{2x} + E_{3x} + \ldots$

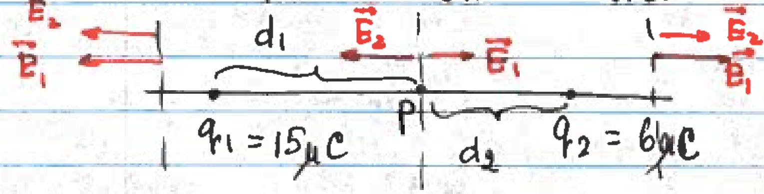
$E_{tot\,y} = E_{1y} + E_{2y} + E_{3y} + \ldots$

Magnitude

$|\vec{E}_{tot}| = E_{tot} = \sqrt{E_{tot\,x}^2 + E_{tot\,y}^2}$

Example 1 : 1D world

Two charges are on a line. Where is the electric field is zero?



q_1, q_2 both positive!
the distance b/w them is $L = 2m$

This point must be b/w two charges

$$E_1 = \frac{kq_1}{d_1^2} \qquad E_2 = \frac{kq_2}{d_2^2}$$

$$\text{if } E_1 = E_2 \quad \frac{q_1}{d_1^2} = \frac{q_2}{d_2^2} \Rightarrow \frac{d_2}{d_1} = \sqrt{\frac{q_2}{q_1}}$$

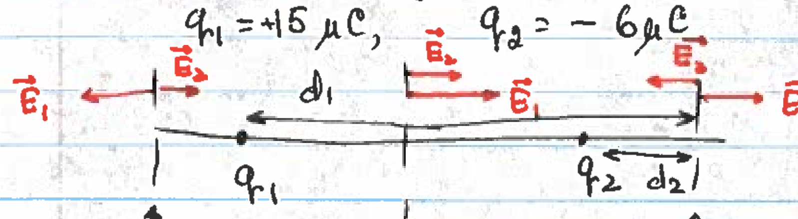
$$d_1 + d_2 = L \Rightarrow d_2 = L - d_1 \Rightarrow \frac{L}{d_1} - 1 = \sqrt{q_2/q_1}$$

$$d_1 = \frac{L}{1 + \sqrt{q_2/q_1}}$$

$$d_2 = \frac{\sqrt{q_2/q_1}}{1 + \sqrt{q_2/q_1}} L$$

If the charges ~~of~~ are opposite:

$q_1 = +15\mu C$, $q_2 = -6\mu C$



suspect

but! $|q_2|$ is smaller and it is farther so $|E_2| < |E_1|$ at any location to the left of q_1

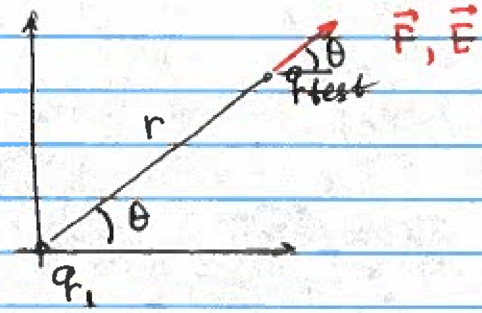
suspect → may work

$$E_1 = E_2$$

$$\frac{kq_1}{d_1^2} = \frac{kq_2}{d_2^2} \qquad d_2 = 2 + d_1$$

A brief reminder how we deal with vector addition. => vector components

Question to consider – how the relative positions of the charges are defined?



$$|\vec{F}| = \frac{kq_1 q_{test}}{r^2}$$

$$|\vec{E}| = \frac{kq_1}{r^2}$$

If we know the angle, it is convenient to use it!

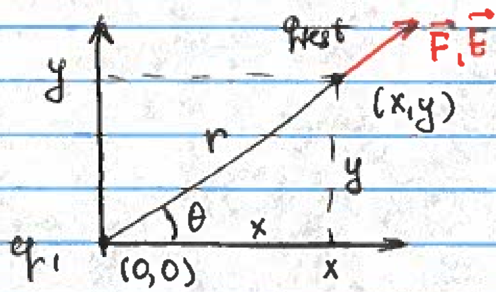
$$F_x = |\vec{F}| \cos\theta, \quad F_y = |\vec{F}| \sin\theta \qquad \vec{F} = F_x \hat{i} + F_y \hat{j}$$

$$E_x = |\vec{E}| \cos\theta, \quad E_y = |\vec{E}| \sin\theta \qquad \vec{E} = E_x \hat{i} + E_y \hat{j}$$

However, if we are given the coordinates sometimes it is easier to use

$$\vec{F} = \frac{kq_1 q_{test}}{r^2} \hat{r} = \frac{kq_1 \cdot q_{test}}{r^3} \vec{r} = \frac{kq_1 q_{test}}{r^3} (x\hat{i} + y\hat{j})$$

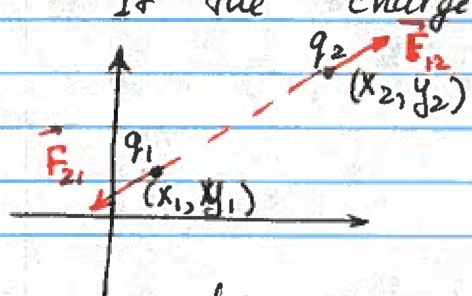
$$F_x = \frac{kq_1 q_{test}}{r^3} x \qquad F_y = \frac{kq_1 q_{test}}{r^3} y \qquad r = \sqrt{x^2 + y^2}$$



These two definitions are equivalent, since

$$\cos\theta = \frac{x}{r} \qquad \sin\theta = \frac{y}{r}$$

If the charge is not in the origin



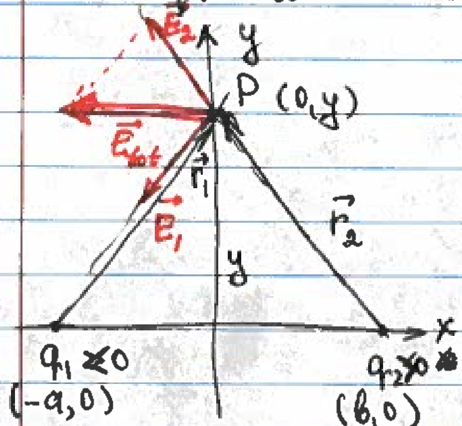
$$F_{12x} = \frac{kq_1 q_2}{((x_1 - x_2)^2 + (y_1 - y_2)^2)^{3/2}} (x_2 - x_1)$$

$$F_{12y} = \frac{kq_1 q_2}{((x_1 - x_2)^2 + (y_1 - y_2)^2)^{3/2}} (y_2 - y_1)$$

$$F_{21x} = \frac{kq_1 q_2}{((x_1 - x_2)^2 + (y_1 - y_2)^2)^{3/2}} (x_1 - x_2) = -F_{12x}$$

$$F_{21y} = \frac{kq_1 q_2}{((x_1 - x_2)^2 + (y_1 - y_2)^2)^{3/2}} (y_1 - y_2) = -F_{12y}$$

Example 2 : 2D world



$$\vec{E}_1 = \frac{kq_1}{r_1^3}\vec{r}_1 = -\frac{k|q_1|}{r_1^3}\vec{r}_1$$

$$r_1 = \sqrt{a^2+y^2} \qquad \vec{r}_1 = a\cdot\hat{i} + y\hat{j}$$

$$\vec{E}_1 = -\frac{k|q_1|}{r_1^3}(a\hat{i} + y\hat{j})$$

$$\vec{E}_2 = \frac{kq_2}{r_2^3}\vec{r}_2$$

$$r_2 = \sqrt{b^2+y^2} \qquad \vec{r}_2 = -b\hat{i} + y\hat{j}$$

$$\vec{E}_2 = \frac{kq_2}{r_2^2}(-b\hat{i} + y\hat{j})$$

$$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2 = \underbrace{\left[-\frac{k|q_1|}{r_1^3}a - \frac{kq_2}{r_2^3}b\right]}_{E_{tot_x}}\hat{i} + \cancel{\left[-\frac{k|q_1|}{r_1^3}\right]}$$

$$+ \underbrace{\left[-\frac{k|q_1|}{r_1^3}y + \frac{kq_2}{r_2^2}y\right]}_{E_{tot_y}}\hat{j}$$

If needed $\quad E_{tot} = |\vec{E}_{tot}| = \sqrt{E_{tot_x}^2 + E_{tot_y}^2}$