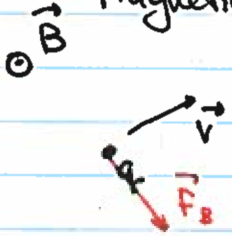
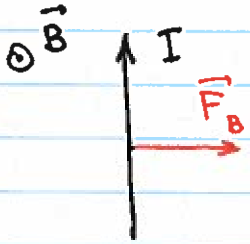
Magnetic force of a current-carrying conductors



Moving charge $\vec{F} = q\,\vec{v} \times \vec{B}$

circular / spiral motion

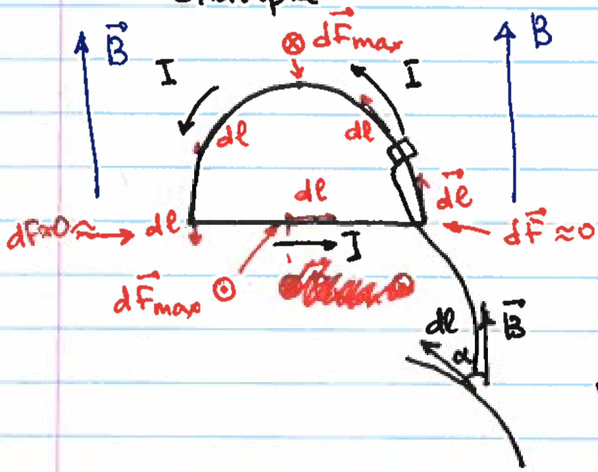


$\vec{F}_B = I \cdot \vec{\ell} \times \vec{B}$

ℓ - the length of the wire in the direction of the current

If the wire is curved $d\vec{F}_B = I \cdot d\vec{\ell} \times \vec{B}$

Example



The magnetic force per unit length is strongest in the straight part and on top

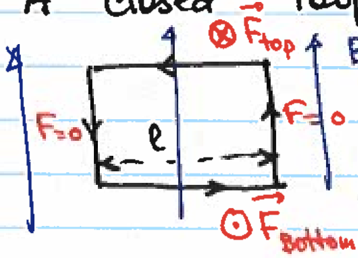
$|dF_{max}| = I\,d\ell \cdot B$

It is zero near the bends

$d\vec{\ell} \parallel \vec{B}$

$|d\vec{F}| = I\,d\ell\,B \sin\alpha < |d\vec{F}_{max}|$

A closed loop in a magnetic field

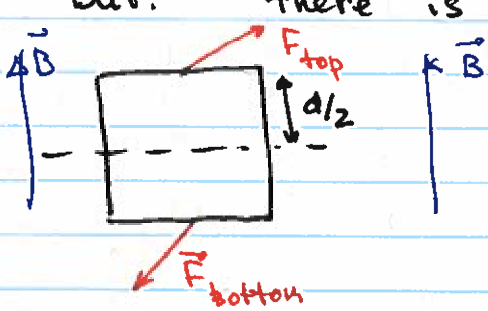


$|\vec{F}_{top}| = |\vec{F}_{bottom}| = \ell \cdot I \cdot B$

but in opposite directions

$F_{tot} = 0$, no net force

But! there is torque!



Each force tries to rotate ~~to~~ the loop in the same direction

$$\tau = F_{top} \cdot \frac{d}{2} + F_{bottom}\,\frac{d}{2} =$$

$$= \ell I B \cdot d = I \underbrace{(\ell \cdot d)}_{\text{area}} \cdot B$$

τ

One can show that for any shape of the flat current loop

$$\vec{\tau} = I \cdot \vec{A} \times \vec{B} = \vec{\mu} \times \vec{B}$$

$\vec{\mu} = I \cdot \vec{A}$ magnetic moment for a current loop

$U_B = -\vec{\mu} \cdot \vec{B}$

Many atoms have internal magnetic moment due to electron(s) orbiting the nucleus. This magnetic moment tends to be proportional to the orbital angular momentum $\vec{L}$: $\vec{\mu} = g_L \vec{L}$

(names s-orbital — angular momentum 0
p-orbital — " — 1
d-orbital — " — 2 ...)

That is why atomic energies change in magnetic field

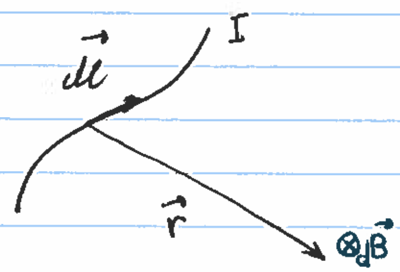
Elementary particles often have intrinsic magnetic moment — spin

The name "spin" comes from incorrect early hypothesis that electron and proton magnetic moment comes from their own rotation, but in reality spin origin is relativistic.

Sources of magnetic field

Magnetic field acts on moving charges and currents $\Longleftrightarrow$ Moving charges and electri currents produce magnetic field

Bio-Savart law



$$\vec{dB} = \frac{\mu_0}{4\pi} I \frac{\vec{dl} \times \hat{r}}{r^2}$$

Magnetic field contribution of current element $\vec{dl}$

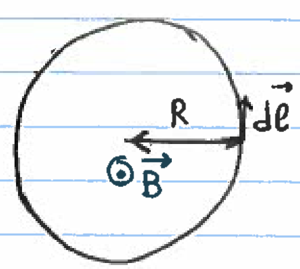
$$\mu_0 = 4\pi \cdot 10^{-7} \frac{T \cdot m}{A}$$

permeability of free space (a constant needed to connect randomly defined SI units)

$$\vec{B} = \int_{\text{along the wire}} \vec{dB} = \frac{\mu_0 I}{4\pi} \boxed{\int \frac{\vec{dl} \times \hat{r}}{r^2}}$$

Geometry of the wire

The calculations are the simplest in the plane of the current, since in this case $\vec{B}$ is perpendicular to this plane



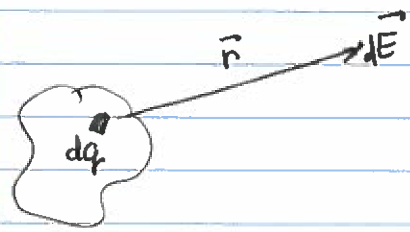
$$\vec{dB} = \frac{\mu_0}{4\pi} I \frac{dl}{R^2} \cdot \hat{z} \text{ (out of the page)}$$

$$\vec{B} = \oint_{\text{along the loop}} \vec{dB} = \frac{\mu_0}{4\pi} I \frac{1}{R^2} \underbrace{\oint dl}_{2\pi R} \hat{z}$$

$$\vec{B} = \cancel{\frac{\mu_0}{2R}} \frac{\mu_0 I}{2R}$$

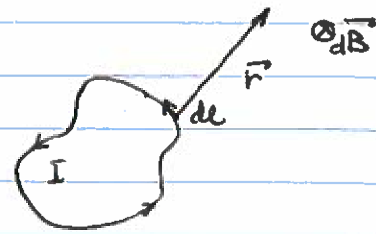
Electrostatic
(time - independent $\vec{E}$)

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{dq(r)}{r^2}\hat{r}$$



Magneto static
(time - independent $\vec{B}$)

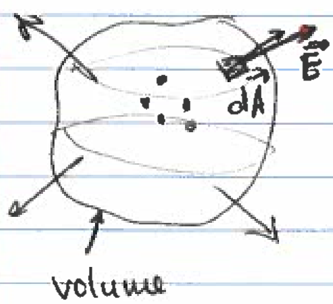
$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \hat{r}}{r^2}$$



General method, often computationally intens

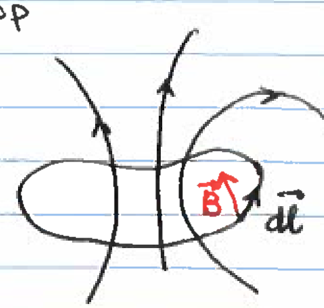
Gauss's Law

$$\Phi_E = \oint_{\substack{\text{closed} \\ \text{volume}}} \vec{E}\, d\vec{A} = \frac{q_{encl}}{\varepsilon_0}$$



Ampere's Law

$$\oint_{\substack{\text{closed} \\ \text{loop}}} \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$



Always valid, occasionally simplifies $\vec{E}$ or $\vec{B}$ calculations ~~for~~ if the system has correct symmetry

Or, more accurately, if one can predict a volume / loop along which we can easily calculate flux of $\vec{E}$ or line integral for $\vec{B}$