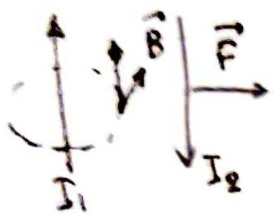
Homework 7

Problem 1

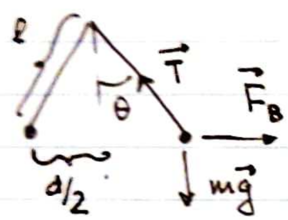
a) The magnetic field created by the first current acts on the second (and vice versa). If the two currents are counter propagating, this force is directed away from the first current



$$B = \frac{\mu_0 I_1}{2\pi d} \qquad F_B = L I_2 B = \frac{\mu_0 I_1 I_2}{2\pi d} L$$

$$F_B/L = \frac{\mu_0 I^2}{2\pi d} \qquad I_1 = I_2$$

b)



Equilibrium: $F_B - T\sin\theta = 0$

$mg - T\cos\theta = 0$

$$\underbrace{F_B = mg\tan\theta}= \frac{\mu_0 I^2}{2\pi d} L$$

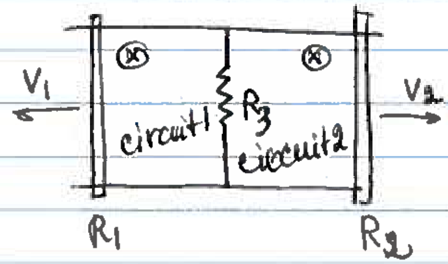
$\theta = 8^\circ$ $\qquad d/2 = \ell \sin\theta$ $\qquad \boxed{d = 2\ell\sin\theta = 1.67\,\text{cm}}$

$$\frac{m}{L} g\tan\theta = \frac{\mu_0 I^2}{2\pi d}$$

$$I = \sqrt{2\pi d g\tan\theta \cdot \frac{m}{L}\frac{1}{\mu_0}} = 67.8A$$

Written assignment 7 (solutions)

Q2.



Circuit 1

$\Delta\Phi_1 = B \cdot d \cdot v_1 \Delta t$

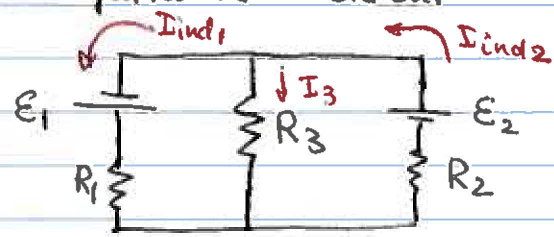
$\mathcal{E}_1 = \left| -\frac{\Delta\Phi_1}{\Delta t} \right| = Bdv_1$

Circuit 2

$\Delta\Phi_2 = B\, d\, v_2 \Delta t$

$\mathcal{E}_2 = \left| -\frac{\Delta\Phi_2}{\Delta t} \right| = Bdv_2$

Equivalent circuit



Kirchhoff's rules

$$\begin{cases} I_2 = I_1 + I_3 & (1) \\ \mathcal{E}_2 - I_3 R_3 - I_2 R_2 = 0 & (2) \\ \mathcal{E}_1 - R_1 I_1 + I_3 R_3 = 0 & (3) \end{cases}$$

$\mathcal{E}_{1,2}$ direction → from Lenz rule to reduce the flux

$I_1 = \frac{\mathcal{E}_1}{R_1} + I_3 \frac{R_3}{R_1}$ from (3)

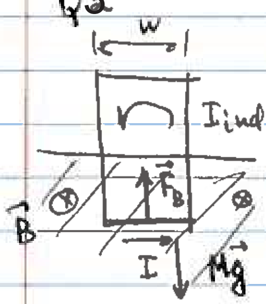
$I_2 = \frac{\mathcal{E}_2}{R_2} - I_3 \frac{R_3}{R_1}$ from (2)

from (1) $\quad \frac{\mathcal{E}_2}{R_2} - I_3 \frac{R_3}{R_2} = \frac{\mathcal{E}_1}{R_1} + I_3 \frac{R_3}{R_1} + I_3$

$$\frac{\mathcal{E}_2}{R_2} - \frac{\mathcal{E}_1}{R_1} = I_3 \left(1 + \frac{R_3}{R_1} + \frac{R_3}{R_2} \right)$$

$$I_3 = \frac{\mathcal{E}_2/R_2 - \mathcal{E}_1/R_1}{1 + \frac{R_3}{R_1} + \frac{R_3}{R_2}} = B \cdot d \, \frac{V_2/R_2 - V_1/R_1}{1 + R_3/R_1 + R_3/R_2}$$

Q3



$$\frac{d\Phi_B}{dt} = Bw \cdot v = |\mathcal{E}_{ind}|$$

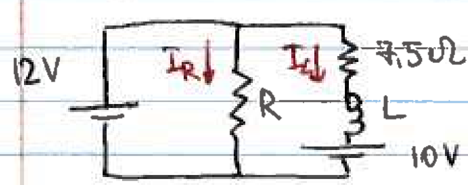
$$I_{ind} = \frac{\mathcal{E}_{ind}}{R} = \frac{Bwv}{R}$$

$$F_B = w \cdot B I_{ind} = \frac{B^2 w^2 v}{R}$$

In the beginning v is small, so the loop accelerates down $F_B - Mg = ma$ $a < 0$ (down)
However, as v increases, F_B approaches Mg, and the velocity increase becomes smaller
For the terminal velocity

$$Mg = F_B = \frac{B^2 w^2 v_t}{R} \Rightarrow v_t = \frac{MgR}{B^2 w^2}$$

Q4



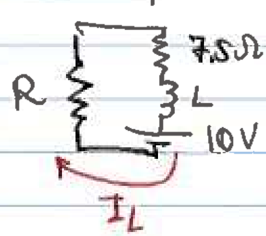
Before disconnect

$I_R = 12V/R$

$12V = I_L \cdot 7.5\Omega + 10V$

$I_L = \frac{2V}{7.5\Omega}$

Once the 12V battery is disconnected, the inductance will prevent I_L to change instantaneously.

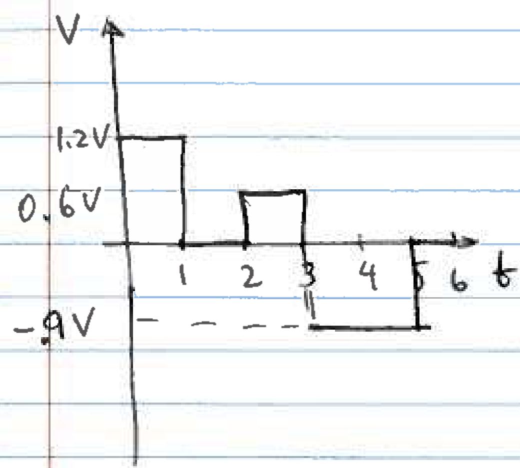


So at the first moment the current through the resistor R is also I_L, and the voltage drop is

$$V_R = I_L \cdot R \Rightarrow R = \frac{V_R}{I_L} = \frac{80V}{2V} \cdot 7.5\Omega$$

$$R_{max} = 300\Omega$$

Q5



0-1s $\frac{dI}{dt} = -6 A/s$ $V = -L\frac{dI}{dt} =$

$= 1.2V$

1-2s $\frac{dI}{dt} = 0$ $V = 0$

2-3s $\frac{dI}{dt} = -3 A/s$ $V = 0.6V$

3-5s $\frac{dI}{dt} = 4.5 A/s$ $V = -0.9V$

5-6s $\frac{dI}{dt} = 0$ $V = 0$