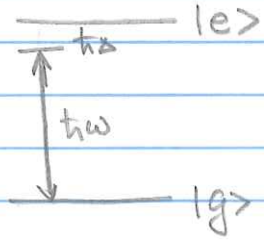
Near-resonant two-level system (Rabi model)



$$\hbar\omega_0 = E_e - E_g$$

$$\Delta = \omega_0 - \omega$$

$$|\psi(t)\rangle = C_g(t)\, e^{-iE_g t/\hbar}|g\rangle + C_e(t)\, e^{-iE_e t/\hbar}|e\rangle$$

[recall $\dot{C}_e(t) = -\frac{i}{\hbar}\sum_k C_k(t)\langle e|\hat{H}_I|k\rangle e^{i\omega_{ek}t}$]

$$\langle g|\hat{H}_I|g\rangle = \langle e|\hat{H}_I|e\rangle = 0 \qquad \langle e|\hat{H}_I|g\rangle = \wp_{eg}E_0\cos\omega t$$

$$\dot{c}_g(t) = -\frac{i}{\hbar}\, c_e\, \wp_{eg}^* E_0^* \cos\omega t\; e^{-i\omega_0 t}$$

$$\dot{c}_e(t) = -\frac{i}{\hbar}\, c_g\, \wp_{eg} E_0 \cos\omega t\; e^{i\omega_0 t}$$

Using RWA (keeping only terms $\sim \omega - \omega_0$)

$$\dot{c}_g(t) = -\frac{i}{2\hbar}\wp_{eg}^* E_0^*\, e^{-i\Delta t}\, c_e(t)$$

$$\dot{c}_e(t) = -\frac{i}{2\hbar}\wp_{eg} E_0\, e^{i\Delta t}\, c_g(t)$$

$$\frac{d}{dt}\left(\dot{c}_e e^{-i\Delta t}\right) = -\frac{i}{2\hbar}\wp_{eg}E_0\,\dot{c}_g(t) = -\frac{|\wp_{eg}E_0|^2}{4\hbar^2}\,\underbrace{e^{-i\Delta t}}\cdot c_e(t)$$

$$\| $$

$$\ddot{c}_e\,\underbrace{e^{-i\Delta t}} - i\Delta\,\dot{c}_e\,\underbrace{e^{-i\Delta t}}$$

$$\ddot{c}_e - i\Delta\,\dot{c}_e + \frac{|\wp_{eg}E_0|^2}{4\hbar^2}\,c_e = 0 \qquad \text{oscillator eqn}$$

<u>Important:</u> I'm using the definition of Rabi frequency different from the book! (mine is more common)

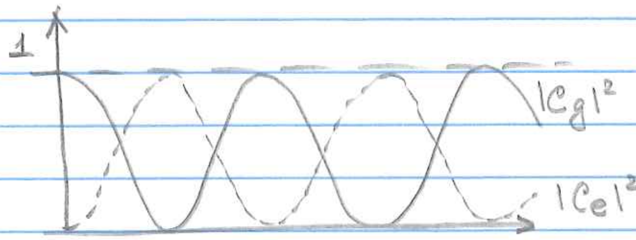
Rabi frequency $\Omega_R = \dfrac{\rho_{eg} E_0}{\hbar}$

$\underline{\Delta = 0}$

$$\left.\begin{aligned}\ddot{C}_e + \tfrac{1}{4}\Omega_R^2 \, C_e = 0 \\ \ddot{C}_g + \tfrac{1}{4}\Omega_R^2 \, C_g = 0\end{aligned}\right] \text{ oscillating}$$

assuming $C_g(t=0) = 1$, $C_e(t=0) = 0$

$$C_g(t) = \cos\tfrac{1}{2}\Omega_R t \qquad C_e(t) = i\sin\tfrac{1}{2}\Omega_R t$$



$$P_e - P_g = \cos^2\frac{\Omega t}{2} - \sin^2\frac{\Omega t}{2} = \cos \Omega_R t$$

For $\Delta \neq 0$

$$C_e(t) = i\frac{\Omega_R}{\sqrt{\Delta^2 + \Omega_R^2}} e^{\frac{i\Delta \cdot t}{2}} \sin\left(\tfrac{1}{2}\sqrt{\Omega_R^2 + \Delta^2}\cdot t\right)$$

$$C_g(t) = e^{i\Delta\cdot t/2}\left\{\cos\left(\tfrac{1}{2}\sqrt{\Omega_R^2+\Delta^2}\cdot t\right) - \frac{i\Delta}{\sqrt{\Delta^2+\Omega_R^2}}\sin\left(\tfrac{1}{2}\sqrt{\Omega_R^2+\Delta^2}\, t\right)\right\}$$

Often $\tilde{\Omega}_R = \sqrt{\Omega_R^2 + \Delta^2}$ is called generalized Rabi frequency.

Notice that Rabi oscillations are fully coherent, since at any moment of time we can define atomic quantum state, i.e, for $\Delta = 0$

$$\psi(t) = \cos\tfrac{1}{2}\Omega_R t \cdot |g\rangle + i\sin\tfrac{1}{2}\Omega_R t \, |e\rangle$$

$$P_g = \cos^2\frac{\Omega_R t}{2} \qquad P_e = \sin^2\frac{\Omega_R t}{2}$$

time - dependent population

(electron oscillates (flops) b/w these two states)

Dressed state picture

A different solution, in which a probability (or population) of the states are time independent => time dependence is only in the phase factor

$$\ddot{c}_e - i\Delta \dot{c}_e + \frac{1}{4}\Omega_R^2 c_e = 0$$

$$c_e = K e^{i\lambda t} \quad \Rightarrow \quad -\lambda^2 + \Delta\cdot\lambda + \frac{1}{4}\Omega_R^2 = 0$$

$$\lambda_{\pm} = \frac{-\Delta \pm \sqrt{\Delta^2 + \Omega_R^2}}{2} = -\frac{\Delta}{2} \pm \frac{1}{2}\tilde{\Omega}_R$$

$$c_e^{(\pm)} = K_{\pm}\, e^{i\lambda_{\pm} t} = K_{\pm}\, e^{-\frac{i\Delta t}{2}}\, e^{\pm i \frac{\tilde{\Omega}_R t}{2}}$$

$$c_g^{(\pm)} = \frac{2i}{\Omega_R} e^{i\Delta t} \dot{c}_e = \frac{2i}{\Omega_R} K_{\pm} e^{i\Delta t}\left(-\frac{i\Delta}{2} \pm \frac{i\tilde{\Omega}_R}{2}\right) e^{-\frac{i\Delta t}{2}}\, e^{\pm i\frac{\tilde{\Omega}_R t}{2}}$$

$$c_g^{(\pm)} = K_{\pm}\frac{\Delta \mp \tilde{\Omega}_R}{\Omega_R}\, e^{i\Delta t/2}\, e^{\pm i\tilde{\Omega}_R t/2}$$

Two eigenstates: $|\psi_{\pm}\rangle$ $(K_{\pm} = 1)$

$$|\psi_{\pm}\rangle = K_{\pm}\left[e^{i\frac{-\Delta \pm \tilde{\Omega}_R}{2}t}\,|e\rangle + \frac{\Delta \mp \tilde{\Omega}_R}{\Omega_R}\, e^{i\frac{\Delta \pm \tilde{\Omega}_R}{2}t}\,|g\rangle\right]$$

Normalization $\langle\psi_{\pm}|\psi_{\pm}\rangle = 1$

$$|K_{\pm}|^2\left(1 + \frac{(\Delta \mp \tilde{\Omega}_R)^2}{\Omega_R^2}\right) = 1 \qquad |K_{\pm}| = \frac{1}{\sqrt{1 + \frac{(\Delta \mp \tilde{\Omega}_R)^2}{\Omega_R^2}}} =$$

$$= \frac{\Omega_R}{\sqrt{\Omega_R^2 + (\Delta \mp \tilde{\Omega}_R)^2}} = \frac{\Omega_R}{\sqrt{\Omega_R^2 + \Delta^2 \mp 2\Delta\tilde{\Omega}_R + \Omega_R^2 + \Delta^2}} = \frac{\Omega_R}{\sqrt{2\tilde{\Omega}_R^2 \mp 2\Delta\tilde{\Omega}_R}} =$$

$$= \frac{\Omega_R}{\tilde{\Omega}_R}\sqrt{\frac{\tilde{\Omega}_R}{2(\Omega_R \mp \Delta)}}$$

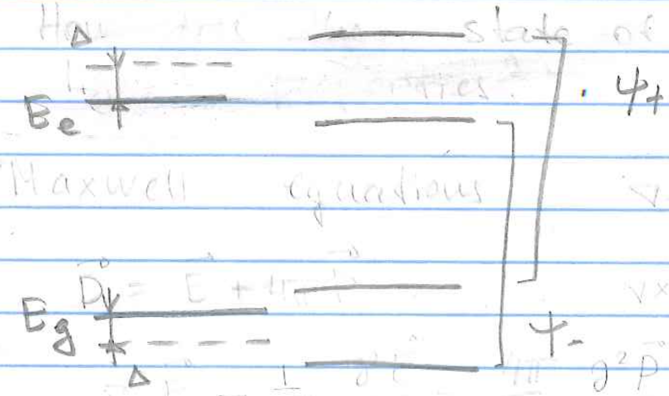
$$|\psi_\pm\rangle = \frac{\Omega_R}{\tilde{\Omega}_R}\sqrt{\frac{\tilde{\Omega}_R}{2(\tilde{\Omega}_R \mp \Delta)}}\left(e^{i\frac{-\Delta \pm \tilde{\Omega}_R}{2}t}|e\rangle + \frac{\Delta \mp \tilde{\Omega}_R}{\Omega_R} e^{i\frac{\Delta \pm \tilde{\Omega}_R}{2}t}|g\rangle\right)$$

The dressed states are stationary, but they are not the eigenstate of the original atomic hamiltonian, atoms are in the quantum superpositions of the states $|e\rangle$ and $|g\rangle$

$$P_e = |\langle e|\psi_\pm\rangle|^2 = \frac{\Omega_R^2}{2(\tilde{\Omega}_R \mp \Delta)\tilde{\Omega}_R}$$

$$P_g = |\langle g|\psi_\pm\rangle|^2 = \frac{\tilde{\Omega}_R \pm \Delta}{2\tilde{\Omega}_R}$$

constant in time



dressed state energies are shifted (dinamic or ac Stark effect)

Can we "see" dress states? yes!

Optical response of the atomic medium — from Maxwell eqns

$$\nabla \times \vec{B} = -\frac{\partial \vec{E}}{\partial t} \qquad \text{plus} \quad \vec{D} = \vec{E} + 4\pi\vec{P}$$

$$\nabla \times \vec{D} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla^2\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = \boxed{\frac{4\pi}{c^2}\frac{\partial^2\vec{P}}{\partial t^2}}$$ atomic medium response

$$\vec{P} = N\langle\vec{d}\rangle$$

$$\langle d\rangle = \langle\psi|-e\vec{r}|\psi\rangle$$ state of the system

$$\langle\psi_\pm|d|\psi_\pm\rangle = \langle\psi_\pm|\,\rho_{eg}|e\rangle\langle g| + \rho_{ge}^*|g\rangle\langle e|\,|\psi_\pm\rangle$$

$$= \rho_{eg}\frac{\tilde{\Omega}_R}{2(\tilde{\Omega}_R \mp \Delta)}\left(\frac{\Delta \mp \tilde{\Omega}_R}{\Omega_R}\right)e^{-i\Delta t} + c.c =$$

$$= \mp\frac{\Omega_R}{2\tilde{\Omega}_R}\rho_{eg}\,e^{i\Delta t} + c.c$$

$$\langle\psi_\pm|d|\psi_\mp\rangle = \left(\frac{\Omega_R}{\tilde{\Omega}_R}\right)^2\frac{\cancel{\Omega_R}}{\sqrt{4(\tilde{\Omega}_R^2-\Delta^2)}}\left(\frac{\Delta\mp\tilde{\Omega}_R}{\Omega_R}e^{-i(\Delta\mp\tilde{\Omega})t}\right.$$

$$\left.-\frac{\Delta\pm\tilde{\Omega}_R}{\Omega_R}e^{i(\Delta\pm\tilde{\Omega}_R)t}\right) =$$

$$= \pm\rho_{eg}\frac{\Omega_R}{2\tilde{\Omega}_R}\sqrt{\frac{\tilde{\Omega}_R\pm\Delta}{\tilde{\Omega}_R\mp\Delta}}\,e^{-i(\Delta\mp\tilde{\Omega}_R)t} -$$

$$\mp\rho_{eg}^*\frac{\Omega_R^*}{2\tilde{\Omega}_R}\sqrt{\frac{\tilde{\Omega}_R\mp\Delta}{\tilde{\Omega}_R\pm\Delta}}\,e^{i(\Delta\pm\Omega')t}$$

Atomic respons at three frequencies

ω, $\omega\pm\tilde{\Omega}_R$

Can be observed in fluorescence in a strong field
(Mollow triplet)

Figure 3:

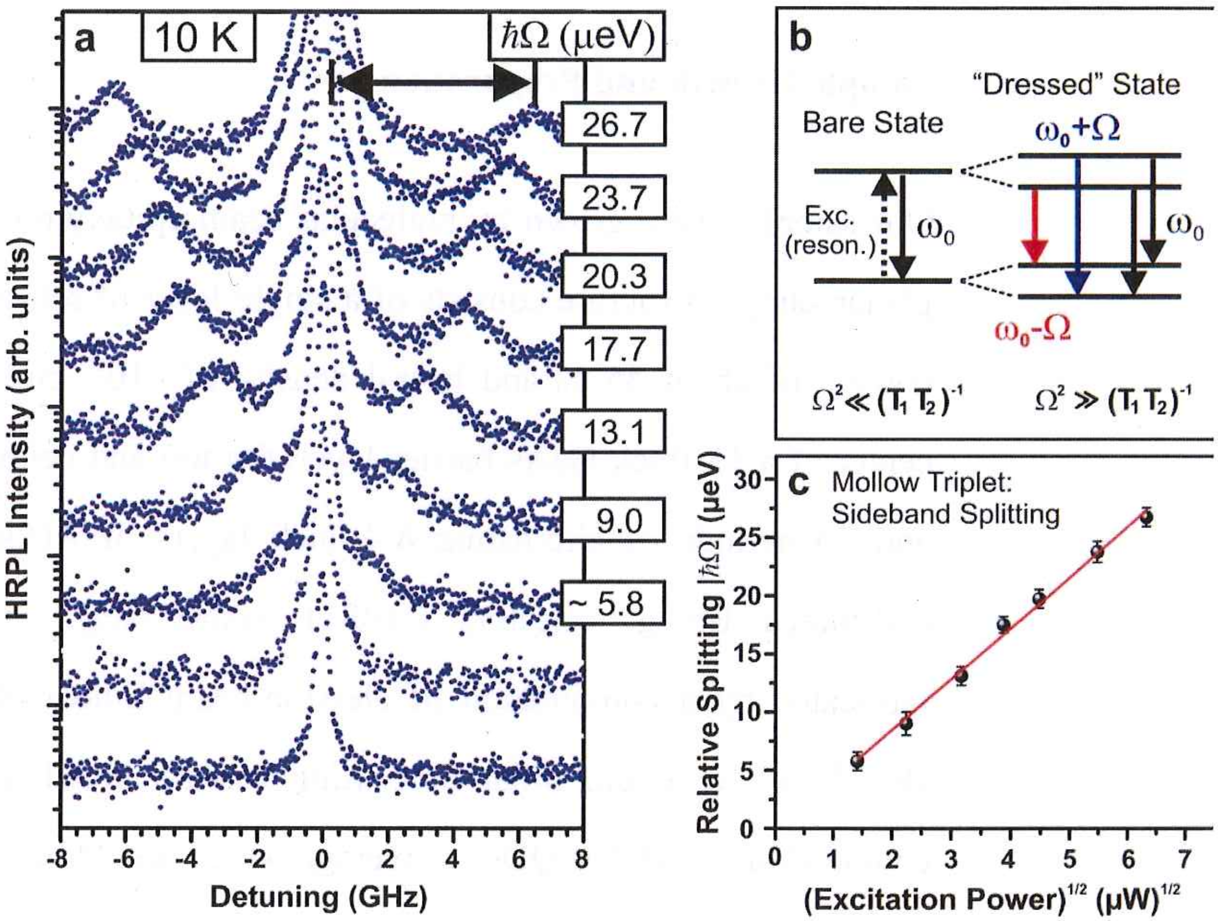


Figure 4: