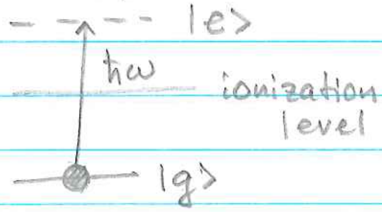
Quantum coherence

Quantized e-m field

$$\hat{\vec{E}}(\vec{r},t) = i \sum_{\vec{k},s} \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}}\, \vec{e}_{\vec{k}s} \left(\hat{a}_{\vec{k}s}\, e^{i\vec{k}\vec{r} - i\omega t} - \hat{a}^{+}_{\vec{k}s}\, e^{-i\vec{k}\vec{r} + i\omega t} \right)$$

To talk about quantum measurements, we need to specify not only quantum object (e-m field) but also the detector

Simplest ideal quantum photodetector



Interaction Hamiltonian

$$H_{EM} = -\vec{d}\,\vec{E}$$

Initial state of the system: $\underset{\text{electrons}}{|g\rangle}\,\underset{\text{photons}}{|i\rangle}$

Final — " — $|e\rangle|f\rangle$

Detectable photocurrent $\propto$ transition probability

$$I_{ph} \propto |\langle e,f|\hat{H}_{EM}|g,i\rangle|^2 = \underbrace{|\langle e|\hat{\vec{d}}|g\rangle|^2}_{\text{atomic properties}} |\langle f|\hat{\vec{E}}|i\rangle|^2$$

(determines quantum efficiency, etc.)

In our model we do not distinguish b/w different final states of em field, thus, we need to sum over all of them

$$I_{ph} \propto \sum_f |\langle e,f|\hat{H}_{EM}|g,i\rangle|^2 \simeq |\langle e|\hat{\vec{d}}|g\rangle| \boxed{\sum_f |\langle f|\hat{\vec{E}}|i\rangle|^2}$$

determines photosignal for a given state $|i\rangle$

Since we assume that the detector must absorb a photon, then only the part of $\hat{E}$ with the annihilation operator will matter.

$$\hat{E}^{(+)} = i\sum_{k,s}\left(\frac{\hbar\omega}{2\varepsilon_0 V}\right)^{1/2} \vec{e}_{\vec{k},s}\ \hat{a}_{\vec{k}s}(\vec{r},t) = \left(\hat{E}^{(-)}\right)^{\dagger}$$

$$\sum_f |\langle f|\hat{E}^{(+)}|i\rangle|^2 = \sum_f \langle i|\hat{E}^{(-)}|f\rangle\langle f|\hat{E}^{(+)}|i\rangle =$$

$$= \langle i|\hat{E}^{(-)}\hat{E}^{(+)}|i\rangle$$ (look very similar to the average intensity)

This equation, however, works only for a pure state. In case of a partially-mixed state, described by the density operator

$$\hat{\rho}_F = \sum_i P_i |i\rangle\langle i|$$

Then, the summation over all possible final states is

$$\sum_i P_i \langle i|\hat{E}^{(-)}\hat{E}^{(+)}|i\rangle = \mathrm{Tr}\left(\hat{\rho}_F \hat{E}^{(-)}\hat{E}^{(+)}\right) = G^{(1)}$$

Quantum analog of intensity

$$G^{(1)}(x,x) = \mathrm{Tr}\left(\hat{\rho}_F\ \underbrace{E^{(-)}(x)\,E^{(+)}(x)}_{\propto\ \hat{a}^{\dagger}\hat{a}}\right) \quad \text{wher } x=(\vec{r},t)$$

$|\psi\rangle = |n\rangle$ number state

$$G^{(1)}(x,x) = \frac{\hbar\omega}{2\varepsilon_0 V} \langle n|\hat{a}^+\hat{a}|n\rangle = \underbrace{n}_{\#\text{ photons}} \cdot \underbrace{\frac{\hbar\omega}{2\varepsilon_0 V}}_{\text{energy of one photon}}$$

$$G^{(1)}(x_1,x_2) = \frac{\hbar\omega}{2\varepsilon_0 V}\, n \cdot e^{i\bar{k}(\bar{r}_2-\bar{r}_1)-i\omega(t_2-t_1)}$$

$$|\gamma^{(1)}(x_1,x_2)| = 1$$

Coherent state $|\psi\rangle = |\alpha\rangle$

$$G^{(1)}(x,x) = |\alpha|^2 \frac{\hbar\omega}{2\varepsilon_0 V} \qquad G^{(1)}(x_1,x_2) = \frac{\hbar\omega}{2\varepsilon_0 V}|\alpha|^2 e^{i\bar{k}(\bar{r}_2-\bar{r}_1)-i\omega(t_2-t_1)}$$

again $|\gamma^{(1)}(x_1,x_2)| = 1$

Pure states are perfectly coherent, since photons are created and absorbed in one mode only (since there are no others)

Repeating the steps for a two-slit interference in a quantum world

$$\hat{E}^{(+)}(x) = K_1 \hat{E}_1^{(+)}(x_1) + K_2 \hat{E}_2^{(+)}(x_2)$$

$$I(\bar{r},t) = Tr\,[\rho_F\, \hat{E}^{(-)}(x)\, \hat{E}^{(+)}(x)] =$$

$$= |K_1|^2\, G^{(1)}(x_1,x_1) + |K_2|^2\, G^{(1)}(x_2,x_2) +$$

$$+ \underbrace{2\,\mathrm{Re}\,[K_1^* K_2\, G^{(1)}(x_1,x_2)]}$$ interference term

where $G^{(1)}(x_1,x_2) = Tr\,[\hat{\rho}_F\, E^{(+)}(x_1)\, E^{(+)}(x_2)]$

First-order quantum coherence function

$$\gamma^{(1)}(x_1,x_2) = \frac{G^{(1)}(x_1,x_2)}{\sqrt{|G^{(1)}(x_1,x_1)\, G^{(1)}(x_2,x_2)|}}$$

Let's examine the functions $G^{(1)}(x,x)$ and $\gamma^{(1)}(x_1,x_2)$ for the case

Single monochromatic plane wave

$$\hat{E}^{(+)} = i\sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}}\; \hat{a}\, e^{i\bar{k}\bar{r} - i\omega t}$$

$$\hat{E}^{(-)} = -i\sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}}\; \hat{a}^{+}\, e^{-i\bar{k}\bar{r} + i\omega t}$$

$$\hat{E}^{(-)}(x_1)\, \hat{E}^{(+)}(x_2) = \frac{\hbar\omega}{2\varepsilon_0 V}\, \hat{a}^{+}\hat{a}\; e^{i\bar{k}(\bar{r}_2 - \bar{r}_1) - i\omega(t_2 - t_1)}$$

$$G^{(1)}(x,x) = \frac{\hbar\omega}{2\varepsilon_0 V}\, Tr\,[\rho\, \hat{a}^{+}\hat{a}] = \frac{\hbar\omega}{2\varepsilon_0 V}\, \langle\psi|\hat{a}^{+}\hat{a}|\psi\rangle$$ for pure state

$$G^{(1)}(x_1,x_2) = \frac{\hbar\omega}{2\varepsilon_0 V}\, \langle\psi|\hat{a}^{+}\hat{a}|\psi\rangle\, e^{i\bar{k}(\bar{r}_2 - \bar{r}_1) - i\omega(t_2 - t_1)}$$