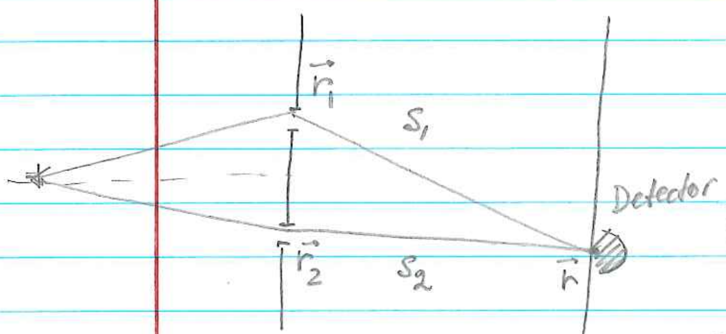
Quantum coherence functions

Classical coherence manifests itself in exhibition of interference.

Two-slit interference



$$\vec{E}(\vec{r},t) = K_1 \vec{E}(\vec{r}_1,t_1) + K_2 \vec{E}(\vec{r}_2,t_2)$$

$$t_{1,2} = t - s_{1,2}/c$$

We assume no change in polarization, so we can treat electric fields as scalars.

K_1, K_2 – geometrical factors, they will take care of any differences b/w two slits, optical paths, ets.

A photodetector records time-averaged light intensite

$$I(\vec{r},t) = \langle |E(\vec{r},t)|^2 \rangle = \lim_{T \gg 1/\omega} \frac{1}{T} \int_0^T |E(\vec{r},t)|^2 dt$$

$$|E(\vec{r},t)|^2 = |K_1|^2 |E(\vec{r}_1,t_1)|^2 + |K_2|^2 |E(\vec{r}_2,t_2)|^2 + 2\,\mathrm{Re}\left(K_1^* K_2\, E^*(\vec{r}_1,t_1) E(\vec{r}_2,t_2)\right)$$

First two terms – independent contributions

$$I_1 = |K_1|^2 \langle |E(\vec{r}_1,t_1)|^2 \rangle$$
$$I_2 = |K_2|^2 \langle |E(\vec{r}_2,t_2)|^2 \rangle$$

The third term is responsible for interference

We can define <u>first-order normalized coherence</u>

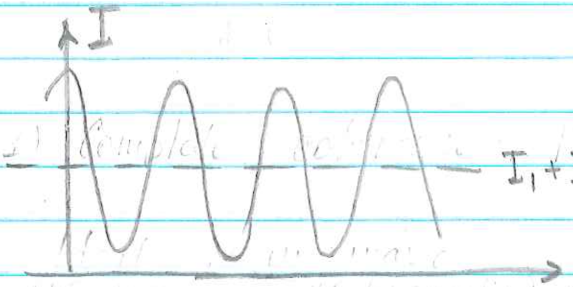
$$\gamma^{(1)}(x_1,x_2) = \frac{\langle E^*(x_1)\, E(x_2)\rangle}{\sqrt{\langle |E(x_1)|^2\, |E(x_2)|^2}} \qquad x_i = \{\vec{r}_i, t_i\}$$

Then we can write

$$I(\vec{r}) = I_1 + I_2 + 2\frac{\sqrt{I_1 I_2}}{|K_1||K_2|}\,\mathrm{Re}\left[K_1^* K_2\, \gamma^{(1)}(x_1;x_2)\right] =$$

$$= I_1 + I_2 + 2\frac{\sqrt{I_1 I_2}}{|K_1||K_2|}\,\mathrm{Re}\left[|K_1| e^{-i\varphi_1}\, |K_2| e^{i\varphi_2}\, |\gamma^{(1)}(x_1,x_2)|\, e^{i\Phi_{12}}\right] =$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\, |\gamma^{(1)}(x_1,x_2)| \cos(\Phi_{12} + \underbrace{\varphi_2 - \varphi_1}_{\text{pathlength difference}})$$



$$I_{max/min} = I_1 + I_2 \pm 2\sqrt{I_1 + I_2}\, |\gamma^{(1)}(x_1,x_2)|$$

Visibility (= constrast of interference friges)

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{I_1 I_2}\, |\gamma^{(1)}(x_1,x_2)|}{I_1 + I_2}$$

For $I_1 = I_2$ $\qquad V = |\gamma^{(1)}(x_1,x_2)|$

Complete coherence $|\gamma^{(1)}(x_1, x_2)| = 1$

Complete decoherence $|\gamma^{(1)}(x_1, x_2)| = 0$

Partial coherence $0 < |\gamma^{(1)}(x_1, x_2)| < 1$

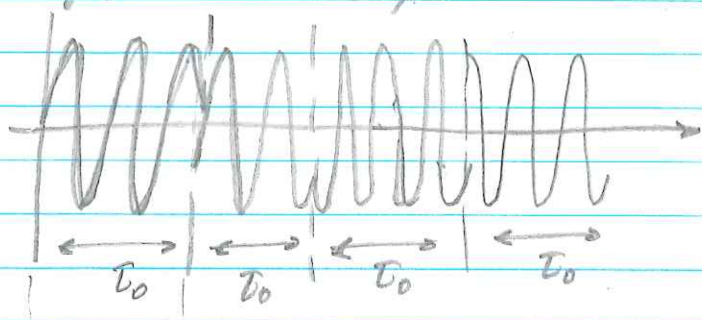
Perfectly monochromatic field

$$E = E_0 \, e^{i\vec{k}\vec{r} - i\omega t}$$

$$\langle E^*(x_1) E(x_2) \rangle = |E_0|^2 \langle e^{i\vec{k}(\vec{r}_1 - \vec{r}_2) - i\omega(t_1 - t_2)} \rangle$$

$$|\gamma^{(1)}(x_1, x_2)| = |\langle e^{i\vec{k}(\vec{r}_1 - \vec{r}_2) - i\omega(t_1 - t_2)} \rangle| = 1$$

More realistically, e-m field maintains its phase for some finite time (coherence time), and then it changes randomly

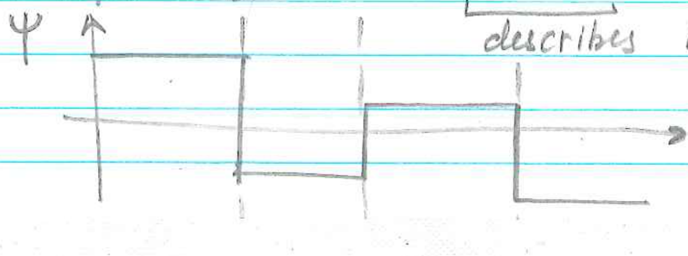


Temporal coherence

$$\langle E^*(x_1) E(x_2) \rangle = \langle E^*(z, t) E(z, t + \tau) \rangle$$

where $\tau = |s_1 - s_2| / c$

$$E(z, t) = E_0 \, e^{ikz - i\omega t} \underbrace{e^{i\psi(t)}}_{\text{describes random phase change}}$$



$$\langle E^*(z,t)\,E(z,t+\tau)\rangle = E_0\, e^{-i\omega\tau} \langle e^{i[\varphi(t+\tau)-\varphi(t)]} \rangle$$

$$\gamma^{(1)}(\tau) = e^{-i\omega\tau} \lim_{T\to\infty} \int_0^T e^{i[\varphi(t+\tau)-\varphi(t)]}\, dt$$

$$= \begin{cases} \left(1-\frac{\tau}{\tau_0}\right) e^{-i\omega\tau} & \tau < \tau_0 \\ 0 & \tau > \tau_0 \end{cases}$$

$|\gamma^{(1)}(\tau)| = 1 - \tau/\tau_0$ for regular phase swithes, More realistically for randomly-distributed phase switches

$$\gamma^{(1)}(\tau) = e^{-i\omega\tau}\, e^{-|\tau|/\tau_0}$$

where τ_0 is the average switch time

$$|\gamma^{(1)}(\tau)| = e^{-|\tau|/\tau_0} \leq 1$$

Interestingly, for short time any light source is coherent and can display interference (sun coherence length $l_{coh} = c \cdot \tau_{coh}$ is about 1 cm, for a laser - many km)