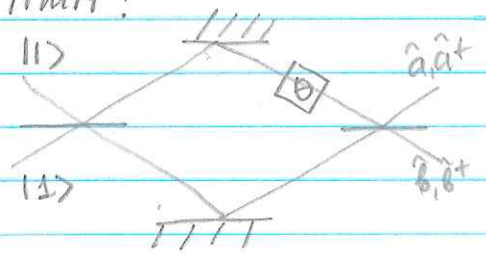
How can we beat the shot-noise limit?



Initial state

$|1\rangle_0 |1\rangle_1$

After the first beamsplitter

$$\frac{1}{\sqrt{2}}\left(|2\rangle|0\rangle + |0\rangle|2\rangle\right)$$

After the beam shifter $\frac{1}{\sqrt{2}}\left(|2\rangle|0\rangle + e^{2i\theta}|0\rangle|2\rangle\right)$

After the second beam-splitter

$$\frac{1}{2\sqrt{2}}\left(1-e^{2i\theta}\right)\left(|2\rangle|0\rangle - |0\rangle|2\rangle\right) + \frac{i}{2}\left(1+e^{2i\theta}\right)|1\rangle|1\rangle$$

Conventional detection does not work!

$\langle \hat{a}^\dagger \hat{a}\rangle = \langle \hat{b}^+ \hat{b}\rangle = 1 \qquad \langle \hat{O}\rangle = 0$

Need cleverer detection method

For example: two-photon detectors (go off only when two photon hits them)

Then we are sensitive only to the first part of the wave function

$$P^{(2ph)}_{1,2} = \frac{1}{4}\left(1-\cos 2\theta\right)$$

$=$ doubling the angle sensitivity

Alternatively, one can measure the <u>parity</u>

of one of the output beams

$$\hat{\Pi}_b = (-1)^{\hat{b}^+\hat{b}} = e^{i\pi \hat{b}^+\hat{b}} \qquad (\hat{\Pi}^2 = \hat{\mathbb{I}})$$

$$\langle \Psi_{fin} | \hat{\Pi}_b | \Psi_{fin} \rangle = \frac{1}{4}|1-e^{2i\theta}|^2 - \frac{1}{4}|1+e^{2i\theta}|^2 =$$

$$= \frac{1}{2}(1-\cos 2\theta) - \frac{1}{2}(1+\cos 2\theta) = -\cos 2\theta$$

$$\Delta \Pi_b = \sqrt{1-\langle \hat{\Pi}_b \rangle^2} = \sin 2\theta$$

$$\Delta\theta = \frac{\Delta \Pi_b}{\left|\left(\frac{\partial \langle \Pi_b \rangle}{\partial \theta}\right)\right|} = \frac{\sin 2\theta}{2\sin 2\theta} = \frac{1}{2} = \frac{1}{\bar{n}}$$

since $\bar{n} = 2$ in this case

Heisenberg limit!