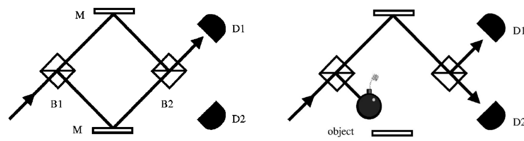
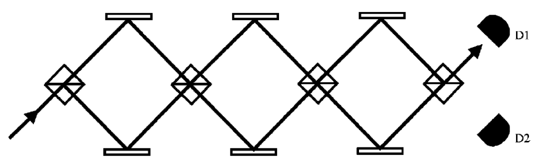
PHYS 481/690

Problem set # 6 *(due November 30)*

A1 *(20 points)*
In class we discussed the scheme for interaction-free measurement based on a Mach-Zender interferometer. It consists of two beam splitters (B1 and B2) and two mirrors (M). Assume that the path length in the upper and lower arms of the interferometer are the same.
(a) First, consider the interferometer when no object blocks either arm. Show that if the reflectivity of the first



beam splitter (R1) is the same as the transmissivity of the second beamsplitter (T2), then a photon entering the interferometer from below will always reach the first detector D1 and never the second (D2).
(b) Suppose a perfectly absorbing object is placed in the lower arm of the interferometer. A photon may reach the first detector as before, but it also has a chance detector. The final possibility is a measurement of the object's presence without interaction, because light never reaches the second detector when the object is not present. Because a photon reaching the first detector gives no information about the presence or absence of an abject, the relative probabilities of the other two possibilities should be compared to evaluate how well the apparatus is performing. Determine the efficiency of the interaction-free measurement, which is defined as the ratio of the probability of interaction-free measurement to the probability of either measurement or absorption as a function of R1. What is the upper limit of the efficiency, and for what value of R1 it occurs?
A2 *(20 points)*
(a) In the improved arrangement arrangement shown below, light is reflected through a series of beam splitters. If the reflectivity of each beam splitter in the series is $R = cos^2(\pi/2N)$, where N is the number of beam splitters, show that light will always reach the first detector (D1).
(b) If an object blocks all of the upper arms of the apparatus, a photon can also be absorbed or reach the second



detector. Determine the efficiency of interaction-free measurement as a function of the number of beam splitters. What is the theoretical limit on the efficiency for this device as N becomes large?
A3 *(10 points)*
When discussing Bell's inequality, we introduced a projection operator for spins:
$\hat{\pi}_\theta = 1/2(1 + \hat{\sigma}_z cos\theta + \hat{\sigma}_x sin\theta)$,
that predicts the output of the spin measurements with a Stern-Gerlach apparatus oriented at the angle θ. Here $\hat{\sigma}_z$ and $\hat{\sigma}_x$ are the Pauli matrices.
For an entangled singlet state
$|\Psi_{1,2}\rangle = 1/\sqrt{2}\,(|\uparrow_1, \downarrow_2\rangle - |\downarrow_1, \uparrow_2\rangle)$
show that
$\langle\Psi_{1,2}|\hat{\pi}_{\theta_a}\hat{\pi}_{\theta_b}|\Psi_{1,2}\rangle = 1/4[1 - cos(\theta_a^{(1)} - \theta_b^{(2)})]$.

A4 *(10 points)*
In class we checked that the entangled singlet state (see problem A3) can violate Bell's inequality. In optics the parametric down conversion process often produces a different Bell state:
$|\Phi_{1,2}\rangle = 1/\sqrt{2}\,(|H_1, H_2\rangle + |V_1, V_2\rangle)$,
where $|H_{1,2}\rangle$ and $|V_{1,2}\rangle$ indicate horizontal and vertical polarizations of the two photons. Using the polarization measurement operator, analogous to the projection operator in the previous problem, verify that such state also violates Bell's inequality.