PHYS 481/690

Problem set # 3 *(due October 5)*

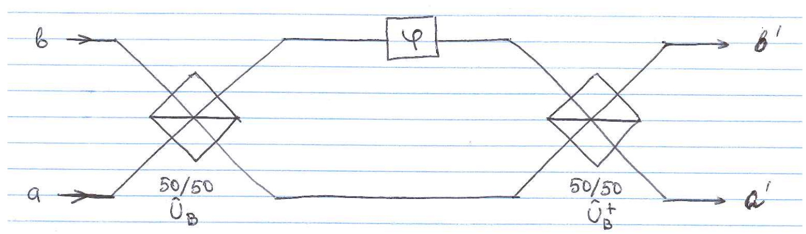
Problems from Gerry& Knight (*Each problem is 10 points.*): 7.18; 5.3; 5.4

A1 *(40 points)* Lossy interferometer
Note: In this problem we will use the beam splitter operator following problem 6.3 from the last assignment, i.e. that the beam splitter operator $\hat{U}_B$, acting with angle θ on modes a and b, is defined as:

$$\hat{U}_B = exp\left[\theta(\hat{a}^\dagger\hat{b} - \hat{a}\hat{b}^\dagger)\right] \tag{1}$$

Let's have a closer look on a Mach-Zehnder interferometer, that consists of two 50/50 beamsplitters, with a phase shifter of variable delay ϕ in one of the arms. In any real interferometer photons can be lost in each arm due to scattering, absorption or some other optical imperfections. The effect of such loss can be modeled by a beamsplitter.

a) Our starting point is the standard Mach-Zehnder interferometer, which uses two 50/50 beamsplitters.

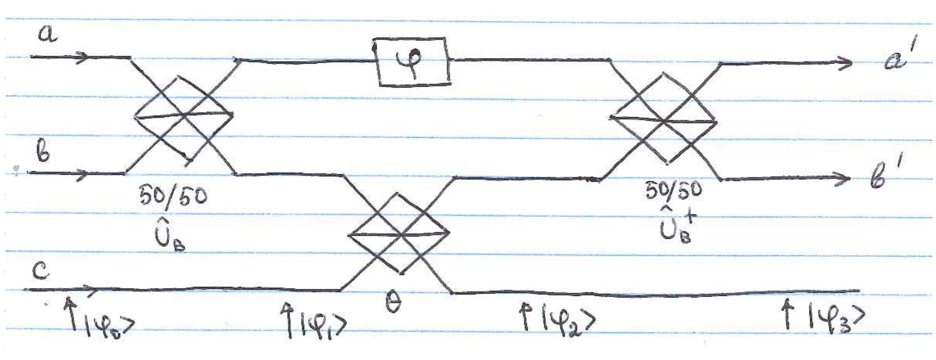


In the absence of loss, and given an input $|0\rangle_a|1\rangle_b$, a photon is measured to be output in mode b with probability $P_b = cos^2(\phi/2)$. The fringe visibility of this output signal is defined as:

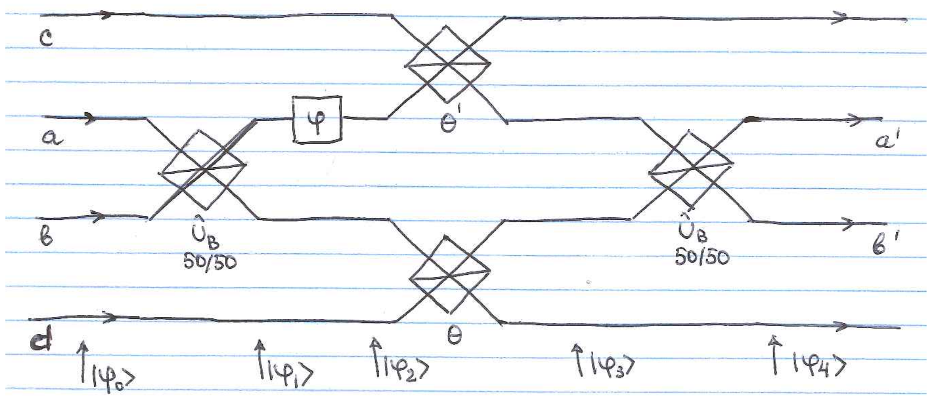
$$V = \frac{P_b^{max} - P_b^{min}}{P_b^{max} + P_b^{min}}, \tag{2}$$

where the min and max are measured by adjusting ϕ. Show that $V = 1$ for this ideal situation.

b) Suppose now that one arm of the interferometer is lossy, and we model this loss with a beam splitter, in this way:



Let the input state be $|\Phi_0\rangle = |0\rangle_a|1\rangle_b|0\rangle_c$. The two outer beamsplitters are the usual 50/50 beamsplitters, while the new intermediate beamsplitter has angle θ. When $\theta = 0$, no photons are lost from b to c, and when $\theta = \pi/2$, any photon in b is moved to c, modeling complete loss. Give the intermediate states $|\Phi_1\rangle$ through $|\Phi_3\rangle$, and compute the probability that a photon is found at the output in mode b, as a function of ϕ and θ. What is the fringe visibility as a function of θ?



c) Now let both arms of the interferometer be lossy, modeled with two beamsplitters (above).
The top beamsplitter has angle θ', and the bottom θ. Give the intermediate states $|\Phi_1\rangle$ through $|\Phi_4\rangle$, and compute the probability that a photon is found at the output in mode b, as a function of ϕ, $\Delta\theta = \theta' - \theta$, and $\theta_{tot} = \theta' + \theta$ What is the fringe visibility as a function of $\Delta\theta$?

d) Continuing the case c), let's now assume, that we count only the cases when at least one photon is measured at the outputs a or b (i.e. we throw away the case when no photons are measured at the output). How does the visibility change, when using this conditional probability? The case when $\Delta\theta = 0$ is known as *balanced loss* Mach-Zehnder interferometer, and your results should show why balancing the loss in two arms is helpful.