Exclusive $\pi^+\pi^-$ Electroproduction

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Abstract

I have studied the effect of the HERMES acceptance on the determination of Legendre moments for exclusive $\pi^+\pi^-$ electroproduction.
Exclusive $\pi^+\pi^-$ production

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Figure 1. Leading twist diagrams for the hard exclusive reaction $e^+T \rightarrow e^+T' \pi^+\pi^-$. Gluon exchange (a) gives rise to pions in the isovector state only, while the quark exchange mechanism (b,c,d) gives rise to pions in both the isoscalar and the isovector state.

- Either 2 gluon or 2 quark exchange
- $p$-like or $f$-like quantum numbers
  - odd $l$
  - even $l$

- Amplitude $\propto \sum_{\ell} a_{\ell} P_{\ell}(\cos \Theta)$

$$\frac{d\sigma_{\pi^+\pi^-}}{d\cos \Theta} \propto \sum_{\ell \ell'} a_{\ell} a_{\ell'} P_{\ell}(\cos \Theta) P_{\ell'}(\cos \Theta)$$

- Look at interference terms $l = 0$, $l' = 1, 3$

$$\frac{d\sigma}{d\cos \Theta} \sim P_{l'}(\cos \Theta) \text{ since } P_0(\cos \Theta) = 1$$

- $P_{\ell}$'s are orthogonal $\implies$

$$\langle P_n \rangle = \frac{\int d\sigma}{\int d\sigma} \int d\cos \Theta \frac{P_n(\cos \Theta) d\cos \Theta}{\int d\sigma}$$

projects out the interference term
\[ \frac{d\sigma}{d\cos(\theta)} = \sum_{\ell \ell'} q_{\ell \ell'} P_{\ell}(\cos(\theta)) P_{\ell'}(\cos(\theta)) \]

\[ P_0(x) = 1 \]
\[ P_1(x) = x \]
\[ P_2(x) = \frac{1}{2}(3x^2 - 1) \]
\[ P_3(x) = \frac{1}{2}(5x^3 - 3x) \]
Figure 4. $m_{\pi\pi}$-dependence of the intensity densities ($P_1(\cos \theta)$), upper panels, and ($P_3(\cos \theta)$), bottom panels, for both hydrogen and deuterium, left and right panels respectively. In the upper panels, the region $0.8 < m_{\pi\pi} < 1.1$ GeV rebinmed in finer channels to better investigate possible contributions from the narrow $f_0(980)$ meson resonance. Also shown are leading twist predictions for the hydrogen target including the two-gluon exchange mechanism contribution, LPSG [4,5] (solid curve at $x = 0.16$). A calculation without the gluon exchange contribution is showed for limited $m_{\pi\pi}$ values, LPPSG [6] (open squares at $x = 0.1$, open triangles at $x = 0.2$). Fig. 1-a. In the above predictions, the contribution from $f_0$ meson decay was not considered. Instead, in the zoomed panel for the hydrogen target, the prediction from [18], which includes the $f_0$ meson contribution, is shown. All experimental data have $<x> = 0.16$ and $<Q^2> = 3$ GeV$^2$. The systematic uncertainty is represented by error bar.

Figure 6. The $x$-dependence of the intensity densities ($P_1(\cos \theta)$) for both targets separately, in the regions $0.30 < m_{\pi\pi} < 0.60$ GeV (left panels) and $0.60 < m_{\pi\pi} < 0.95$ GeV (right panels). Theoretical predictions from LPPSG [6] (stars) for hydrogen are compared with the data. In these computations, the two-gluon exchange mechanism contribution to the process is neglected. The systematic uncertainty is given by the error band.
• Cross check of analysis done by Sasha Borissov

• Monte Carlo simulation showed with large statistical errors that acceptance corrections to $\langle P_1 \rangle$ and $\langle P_3 \rangle$ were probably not important.

$$\langle P_n \rangle = \frac{1}{N} \sum_{i=1}^{N} P_n(\cos \Theta_i)$$ measured values

$\Rightarrow$ cannot make acceptance corrections to a spectrum and then fit

$\Rightarrow$ How wrong might $\langle P_n \rangle$ be due to holes in 4π acceptance?

This is a general problem also applicable to SSA $\langle \sin \phi \rangle$ moments: $\frac{1}{N} \sum_{i} \sin \Theta_i$

• Create a toy Monte Carlo to study the effects of acceptance on moments $\langle P_n \rangle$
Toy M.C. \[ \text{cp} \to \text{ep} (\pi^+ \pi^-) \]
\[ \text{ed} \to \text{ed} (\pi^+ \pi^-) \]

Exclusive

Describe kinematics with 4 variables:
\[ \{x, Q^2, m_{\pi^+ \pi^-}, P_L\} \]

For each quadruplet, we can ask what is the \(\pi^+ \pi^-\) detection efficiency and measured \(<p_1><p_3>\) moments.

**Choose at random**

- \(\phi \) azimuthal angle of \( \vec{q} \)
- \(\phi_L\) azimuthal angle of \( \vec{P_L}\) around \( \vec{q} \)
- \(\phi_{cm}\) azimuthal angle of \( \pi^+\) emission around direction of \( \pi^+ \pi^- \) momentum.

Choose \( \Theta_{cm}\) direction of \( \pi^+ \) in \( \pi^+ \pi^- \) CM frame with \(-\vec{z}\) the recoil direction of \( N \).

Distributions

- Flat: \( f(x) = \frac{1}{2} \) on \([-1, 1]\)
- Triangular: \( f(x) = \frac{1}{2}(x+1) \)
- Quadratic: \( f(x) = \frac{3}{2}x^2 \)
- Cubic: \( f(x) = \frac{1}{2}(1+x^3) = \alpha_0 p_0 + \alpha_1 p_1 + \alpha_2 p_3 \)
Do isodipole CM distributions 
generate non-zero $<p_t>$ and $<p_z>$ 
due to holes in acceptance?

$<p_t>$

NO!

$<p_z>$

Efficiency for $e^+e^-$ detection in HERMES
Expected $<q_1>$ for thrown distribution.

OK down to about 50% efficiency.

Expected $<p_3>$.
6 bins in $x$ for $0.3 < m_{\pi\pi} < 0.6$ GeV

6 bins in $x$ for $0.6 < m_{\pi\pi} < 0.95$ GeV

11 bins in $m_{\pi\pi}$ for $x > 0.1$

INPUT $\langle x \rangle$, $\langle Q^2 \rangle$, $\langle m_{\pi\pi} \rangle$ for each bin $\langle p_T \rangle$

In general, extracted $\langle p_T \rangle$ is far from expected value

Full averages over a bin will likely come closer to ideal value... but not completely,
Efficiency at average kinematic variables:

$\langle x \rangle = 0.17$

$\langle Q^2 \rangle = 3.2 \text{ GeV}^2$

$\langle m_{\pi^+} \rangle = 0.52 \text{ GeV}$

$P_T$

$P_T$ distribution for event sample (varies little over $m_{TT}$)
Deuterium $0.6 < m_{\pi\pi} < 0.95$ GeV

$\Delta E = \frac{M_X^2 - M^2}{2M}$

Counts vs. delta E

Cos $\theta$ distribution

X: $\theta$ is z-axis
*
*: N recoil is $-z$ axis, our choice

Counts vs. cos $\theta$
>50% acceptance for 2-pion events

\[ z = 4.38x - 0.85 \]
\[ Q^2 = 25.6x - 1.28 \]
\[ x = 0.05 \]
\[ Q^2 = 1 \]

\[ \gamma = 0.21 \]

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Population in bottom is the same for each point in the \( x-Q^2 \) plot

\[ P_T = 5.26 m_{\pi\pi} - 2.26 \]

\[ \uparrow 63 \]

\[ \downarrow 0.37 \]
Green: with 250% efficiency cuts
Red: all data

$\angle p_{37}$

$\angle p_{17}$

deuteron

$M_{WW}$

proton

$M_{TT}$
Deuteron

$\Delta p, 7$

$\sqrt{s} = 0.5$

$\sqrt{s} = 0.8$

$\sqrt{s} = 1.11$

$x = \Delta x$
Conclusions

- Toy Monte Carlo is a nice way to get an understanding of $<p_n>$ within HERMES acceptance.

- Present estimations of errors in $<p_n>$ due to acceptance are small compared to the statistical error bars.

- Exclusive $\pi^+\pi^-$ analysis and paper are OK as they presently stand.

- Any future analysis with improved statistics will need to reckon with acceptance corrections to $<p_n>$.