BoNuS Proton Momentum Calibration Using Quasi-Elastic Events

BoNuS Technical Note

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1 The Spectator Model

In the spectator model we assume that for electron scattering from a neutron in deuterium the proton is on-shell before scattering and there is no final-state interaction between the proton and neutron. Therefore, the measured spectator proton momentum $\vec{p}$ is the proton’s internal momentum before scattering as well. The neutron’s momentum before scattering is therefore $-\vec{p}$ since the deuteron is at rest.

Let $M$ be the proton mass.
Let $M_d = 2M$ be the deuteron mass.
Let $q = (\nu, \vec{q})$ be the 4-momentum transfer of the electron to the neutron in the lab frame.
Let $Q^2 = -q \cdot q$.
Let $p_p = (E_p, \vec{p})$ be the 4-momentum of the spectator proton in the lab frame.
Let $p = |\vec{p}|$.
Let $p_n = (E_n, -\vec{p})$ be the 4-momentum of the neutron before scattering in the lab frame.
Let $x = \frac{-q \cdot q}{2p_n \cdot q}$ be the true momentum fraction.
Let $x_B = \frac{Q^2}{2M \nu}$.
Let $\theta$ be the angle between $\vec{p}$ and $\vec{q}$.
Let $a = 2 - x_B + \frac{2M}{\nu}$.
Let $b = 1 + \frac{2M}{\nu}$.
Let $c = \frac{\nu}{p} \cos \theta$.
Let $\alpha = \sqrt{p^2 + M^2 - cp}$.

Then,

$$M_d = 2M = E_n + E_p \quad \text{and} \quad E_p = \sqrt{p^2 + M^2}.$$  

For quasi-elastic scattering

$$M^2 = (p_n + q)^2 = 5M^2 - Q^2 - 4ME_p + 4M\nu - 2E_p\nu + 2\vec{p} \cdot \vec{q}. \quad (1)$$  

Dividing Eq. 1 by $2M \nu$ and rearranging the terms yields
\[ 2 - x_B = \alpha + \frac{2}{\nu}(E_p - M) \]  

(2)

From the definition of \( x \) we find that

\[ x = \frac{x_B}{2 - \alpha}. \]  

(3)

We would expect that \( x = 1 \) for quasi-elastic scattering, which would imply that \( 2 - x_B = \alpha \).

However, Eq. 2 shows that this is only true in the limit as \( \nu \to \infty \).

If we assume that we know \( \theta \) fairly well from the BoNuS detector, we can solve for the spectator proton momentum \( p \) using Eq. 2:

\[ \frac{p}{M} = \frac{ac \pm \sqrt{b^2(a^2 - b^2 + c^2)}}{b^2 - c^2}. \]  

(4)

## 2 Procedure

1) Select quasi-elastic events with a cut \( x_B > 0.8 \).
2) Determine \( \theta \) from BoNuS and CLAS.
3) Using \( x_B, \nu, |\vec{q}| \) and \( \theta \), calculate \( a, b \) and \( c \).
4) Calculate \( p \) from the positive solution in Eq. 4.
5) Compare \( p \) with the value given by BoNuS tracking.

## References