Homework 11

Prerequisites: Read chapter 10.

Problem 1 (5 points):

If we know inertia tensor calculated with respect to the center of mass, i.e. I_{ik}, show that inertia tensor of this body calculated with respect to the point shifted by $\vec{a}$ from the CM (i.e. $\vec{r}\,' = \vec{r} - \vec{a}$ is given by the formula

$$I'_{ik} \;=\; I_{ik} + M(a^2\delta_{ik} - a_i a_k) \tag{1}$$

where M is the total mass of the body.

Problem 2 (5 points):

Calculate the inertia tensor of an uniform sphere with respect to the center of mass. Show all components of the tensor. The sphere has mass M and radius R.

Now calculate the inertia tensor of the same sphere displaced from the CM by vector $\vec{d} = R\hat{x}$. Show all components of I_{ik}.

Problem 3 (5 points):

In class, we discussed dynamics of a cylinder rolling inside a large fixed cylindrical surface. Now consider two different situations in which different type of cylinders roll inside of the same fixed cylindrical surface with the radius R. First, a cylinder with mass M, outer radius a, and an axial cylindrical hole with radius $a/2$. Second, a cylinder has mass $2M$ and radius $2a$; this cylinder has no holes. Find the ratio of periods of rolling back and force motion for this two cases.

Problem 4 (5 points):

Consider the isolated Moon-Earth system. Due to the friction between tidal motion and more solid parts of the Earth, our planet is slowing down its rotation. Find the final distance between Earth and Moon centers.

For simplicity, assume that the Moon orbit is a perfect circle around the CM of Earth, and that the Earth and the Moon are perfect spheres.

Note: the Earth will not stop its rotation. Due to the tidal lock both the Currently, the Moon is in the tidal lock and faces the Earth with the same side. But eventually the Moon and the Earth will face each other with the same sides. Yep, half of the Earth will be unlucky and will not see the Moon.

Mass of the Earth $M_e = 5.972 \times 10^{24}$ kg, mass of the Moon $M_m = 7.34 \times 10^{22}$ kg, current distance between then Moon and the Earth $D = 384.4 \times 10^6$ m, radius of the Earth $R_e = 6.371 \times 10^6$ m, radius of the Moon $R_m = 1.737 \times 10^6$ m.