Homework 02

Prerequisites: Read chapter 8.

Problem 1 (2 points):

We derived that orbits in the attractive potential $-\gamma/r$ described by the following formula.

$$\frac{c}{r} = 1 + e\cos\phi \tag{1}$$

Show that for $0 \le e < 1$, the orbit has elliptical form. I.e. it satisfies

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{2}$$

where x and y are counted from the orbit symmetry axes intersection, a is semimajor axis, and b is semiminor axis. Note that the origins are different in eqs. (1) and (2), see figure 8.10 in the book.

Problem 2 (4 points):

Derive expressions for a and b in eq. (2). Express them via c and e.

Problem 3 (2 points):

Show that area of the ellipse described by eq. (2) is equal to πab.

Problem 4 (4 points):

During the class, we derived

$$\int d\phi = \int \frac{\frac{l}{\mu r^2} dr}{\sqrt{\frac{2}{\mu}\left(E - \frac{l^2}{2\mu r^2} + \frac{\gamma}{r}\right)}} \tag{3}$$

Show that you can arrive to eq. (1) by proper integration of the right hand side. It might be useful to use a new variable $x = 1/r$

Problem 5 (8 points):

Using angular momentum and energy conservation, find expression for tangential component of velocity at the closest (and farthest) point to the CM. Express final answer via the gravitational constant (G), semimajor axis (a), total mass (M), and the eccentricity (e).

What is the ratio of the tangential components taken as speed at the closest point over the speed at the farthest point?