Chapter 1

Single and Double-Slit Interference, One Photon at a Time

Experiment objectives: Study wave-particle duality for photons by measuring interference pattern in the Young double-slit experiment using conventional light source (laser) and a single-photon source (strongly attenuated lamp).

History

There is a rich historical background behind the experiment you are about to perform. Isaac Newton first separated white light into its colors, and in the 1680's hypothesized that light was composed of 'corpuscles', supposed to possess some properties of particles. This view reigned until the 1800's, when Thomas Young first performed the two-slit experiment now known by his name. In this experiment he discovered a property of destructive interference, which seemed impossible to explain in terms of corpuscles, but is very naturally explained in terms of waves. His experiment not only suggested that such 'light waves' existed; it also provided a result that could be used to determine the wavelength of light, measured in familiar units. Light waves became even more acceptable with dynamical theories of light, such as Fresnel's and Maxwell's, in the 19th century, until it seemed that the wave theory of light was incontrovertible.

And yet the discovery of the photoelectric effect, and its explanation in terms of light quanta by Einstein, threw the matter into dispute again. The explanations of blackbody radiation, of the photoelectric effect, and of the Compton effect seemed to point to the existence of 'photons', quanta of light that possessed definite and indivisible amounts of energy and momentum. These are very satisfactory explanations so far as they go, but they throw into question the destructive-interference explanation of Young's experiment. Does light have a dual nature, of waves and of particles? And if experiments force us to suppose that it does, how does the light know when to behave according to each of its natures?

It is the purpose of this experimental apparatus to make the phenomenon of light interference as concrete as possible, and to give you the hands-on familiarity which will allow you

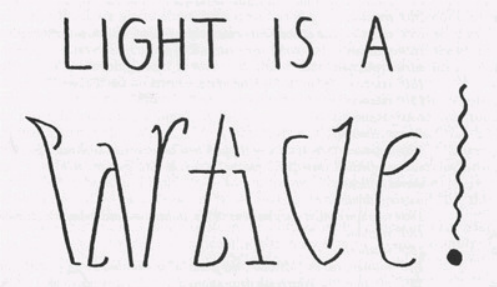

Figure 1.1: "Light is a Particle / Light is a Wave" oscillation ambigram (from *For the Love of Line and Pattern*, p. 30).

to confront wave-particle duality in a precise and definite way. When you have finished, you might not fully understand the mechanism of duality – Feynman asserts that nobody really does – but you will certainly have direct experience of the actual phenomena that motivate all this discussion.

Experimental setup

Equipment needed: Teachspin "Two-slit interference" apparatus, oscilloscope, digital multimeter, counter.

Important: before plugging anything in, or turning anything on confirm that the shutter (which protects the amazingly sensitive single-photon detector) is **closed**. Locate the detector box at the right end of the apparatus, and find the rod which projects out of the top of its interface with the long assembly. Be sure that this rod is pushed all the way down; take this opportunity to try pulling it vertically upward by about 2 cm, but then ensure that it's returned to its fully down position. Also take this occasion to confirm, on the detector box, that the toggle switch in the HIGH-VOLTAGE section is turned off, and that the 10-turn dial near it is set to 0.00, fully counter-clockwise.

To inspect the inside of the apparatus open the cover by turning four latches that hold it closed. The details of the experimental apparatus are shown in Fig. 1.2. Take time to locate all the important components of the experiment:

- Two distinct light sources at the left end: one a red *laser* and the other a green-filtered *light bulb*. A toggle switch on the front panel of the light source control box switches power from one source to the other.

- Various *slit holders* along the length of the long box: one to hold a two-slit mask, one for slit blocker, and one for a detector slit. Make sure you locate *slits* (they may be installed already) and two *micrometer drives*, which allow you to make mechanical adjustments to the two-slit apparatus. **Make sure you figure out how to read the micrometer dials!** On the barrel there are two scales with division of 1 mm, shifted with respect to each other by 0.5 mm; every fifth mark is labeled with an integer 0,

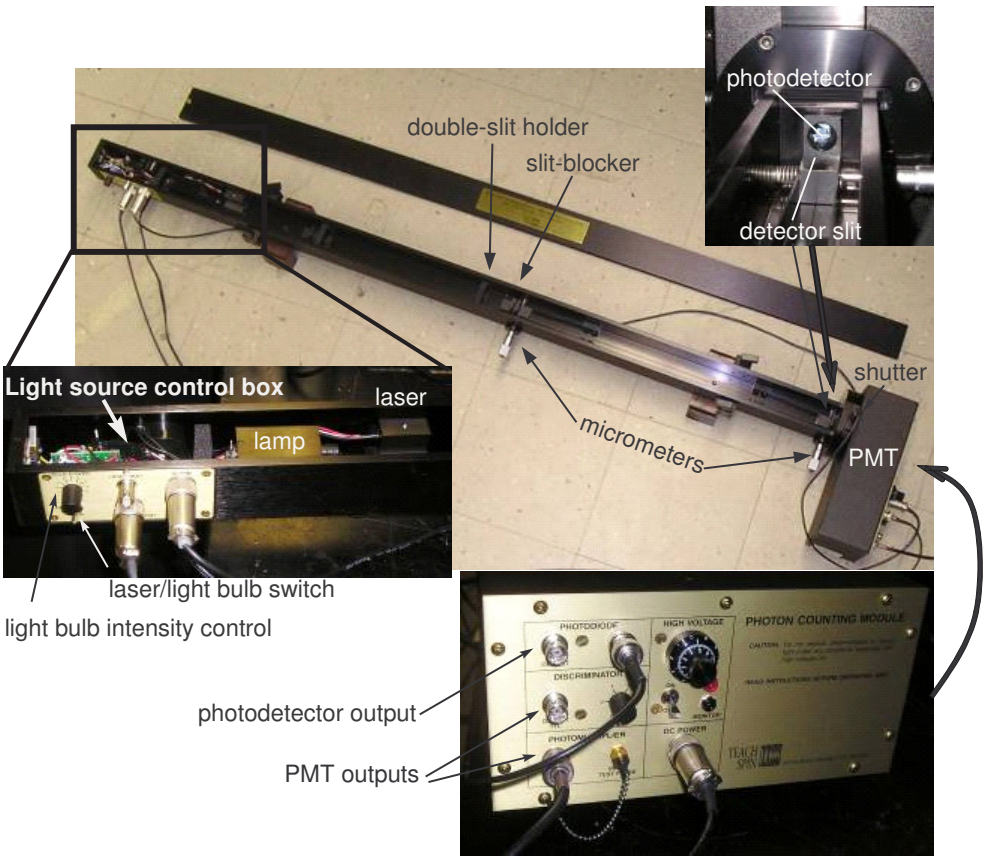

Figure 1.2: The double slit interference apparatus.

5, 10 and so on: these are at 5-mm spacing. The complete revolution of the drum is 0.5 mm, and the smallest division on the rotary scale is 0.01 mm.

- Two distinct light detectors at the right-hand end of the apparatus: a *photodiode* and a *photomultiplier tube* (PMT for short). The photodiode is used with the much brighter laser light; it's mounted on light shutter in such a way that it's in position to use when the shutter is closed (pushed down). The photomultiplier tube is extremely sensitive detector able to detect individual photons (with energy of the order of 10^{-19} J, and it is used with the much dimmer light-bulb source. Too much light can easily damage it, so **PMT is safe to use only when the cover of the apparatus is in place, and only when the light bulb is in use**. It is exposed to light only when the shutter is in its up position.

Experimental procedure

The experiment consists of three steps:

1. You will first observe two-slit interference directly by observing the intensity distribution of a laser beam on a viewing screen.

2. Using the photodiode you will accurately measure the intensity distribution after single- and two-slit interference patterns, which can be compared to predictions of wave theories of light.
   These two steps recreate original Young's experiment.

3. Then using a very weak light source you will record the two-slit interference pattern one photon at a time. While this measurement will introduce you to single-photon detection technology, it will also show you that however two-slit interference is to be explained, it must be explained in terms that can apply to single photons.

Visual observation of a single- and two-slit interference

For this mode of operation, you will be working with the cover of the apparatus open. Switch the red diode laser on using the switch in the light source control panel, and move the laser in the center of its magnetic pedestal so that the red beam goes all the way to the detector slit. The diode laser manufacturer asserts that its output wavelength is 670 ± 5 nm, and its output power is about 5 mW. ***As long as you don't allow the full beam to fall directly into your eye, it presents no safety hazard.*** Place a double slit mask on the holder in the center of the apparatus, and then put your viewing card just after the mask to observe the two ribbons of light, just a third of a millimeter apart, which emerge from the two slits. Move you viewing card along the beam path to see the interference pattern forming. By the time your viewing card reaches the right-hand end of the apparatus, you'll see that the two overlapping ribbons of light combine to form a pattern of illumination displaying the celebrated "fringes" named after Thomas Young.

Position a viewing card at the far-right end of the apparatus so you can refer to it for a view of the fringes. Now it is the time to master the control of the slit-blocker. By adjusting the multi-turn micrometer screw, make sure you find and record the ranges of micrometer reading where you observe the following five situations:

1. both slits are blocked;

2. light emerges only from one of the two slits;

3. both slits are open

4. light emerges only from the other slit;

5. the light from both slits is blocked.

It is essential that you are confident enough in your ability to read, and to set, these five positions that you'll be able to do so even when the box cover is closed. In your lab book describe what you see at the viewing card at the far-right end of the apparatus for each of the five settings.

One slit is open: According to the wave theory of light, the intensity distribution of light on the screen after passing a single slit is described by Fraunhofer diffraction (see Fig. 1.4 and the derivations in the Appendix):

$$I(x) = I_0 \frac{(\sin(\frac{\pi a}{\lambda}\frac{x}{\ell}))^2}{(\frac{\pi a}{\lambda}\frac{x}{\ell})^2}, \tag{1.1}$$

where I is the measured intensity in the point x in the screen, I_0 is the intensity in the brightest maximum, a is the width of the slit, and ℓ is the distance between the slit and the screen (*don't forget to measure and record this distance in the lab journal!*)

In your apparatus move the slit blocker to let the light go through only one slit and inspect the light pattern in the viewing screen. Does it looks like the intensity distribution you expect from the wave theory? Take a minute to discuss how this picture would change if the slit was much wider or much narrower.

Two slits are open: Now move the slit blocker to the position that opens both slits to observe Young's two-slit interference fringes. Again, compare what you see on the screen with the interference picture predicted by wave theory:

$$I(x) = 4I_0 \cos^2\left(\frac{\pi d}{\lambda}\frac{x}{\ell}\right)\left[\frac{\sin(\frac{\pi a}{\lambda}\frac{x}{\ell})}{\frac{\pi a}{\lambda}\frac{x}{\ell}}\right]^2, \tag{1.2}$$

where an additional parameter d is the distance between centers of the two slits. Discuss how this picture would change if you vary the width and the separation of the two slits, and the wavelength of the laser. Make a note of your predictions in the lab book.

Quantitative characterization of interference patterns using laser light

At this stage you will use a photodiode to measure the intensity distribution of the interference pattern by varying the position of the detector slit. You will continue using the red laser. While you may conduct these measurements with the box cover open, room light will inevitably add some varying background to your signals, so it is a good idea to dim the room lights or (even better!) to close up the cover of the apparatus. For convenience, have the slit-blocker set to that previously determined setting which allows light from both slits to emerge and interfere.

The shutter of the detector box will still be in its closed, or down, position: this blocks any light from reaching the PMT, and correctly position a 1-cm^2 photodiode, which acts just like a solar cell in actively generating electric current when it's illuminated. The output current is proportional to total power illuminating the detector area, so it is important to use a narrow slit allow only a selected part of the interference pattern to be measured. Make sure that a detector slit mask (with a single narrow slit) on a movable slit holder at the right-hand side of the apparatus is in place. By adjusting the micrometer screw of the detector slit, you can move the slit over the interference pattern, eventually mapping out its intensity distribution quantitatively. For now, ensure that the detector slit is located somewhere near the middle of the two-slit interference pattern, and have the slit-blocker set to the setting which allows light from both slits to emerge and interfere.

The electric *current* from the photodiode, proportional to the *light intensity*, is conducted by a thin coaxial cable to the INPUT BNC connector of the photodiode-amplifier section of the detector box, and converted to *voltage* signal at the OUTPUT BNC connector adjacent to it. Connect to this output a digital multimeter set to 2 or 20-Volt sensitivity; you should see a stable positive reading. Turn off the laser first to record the "zero offset" - reading of the multimeter with no light. You will need to subtract this reading from all the other reading you make of this output voltage.

Turn your laser source back on, and watch the photodiode's voltage-output signal as you vary the setting of the detector-slit micrometer. If all is well, you will see a systematic variation of the signal as you scan over the interference pattern. Check that the maximum signal you see is about 3-8 Volts; if it is much less than this, the apparatus is out of alignment, and insufficient light is reaching the detector.

Initial tests of wave theory of light: If we assume that the light beam is a stream of particle, we would naively expect that closing one of two identical slits should reduce the measured intensity of light at any point on the screen by half, while the wave theory predicts much more dramatic variations in the different points in the screen. Which theory provide more accurate description of what you see?

- Find the highest of the maxima – this is the "central fringe" or the "zeroth-order fringe" which theory predicts, – and record the photodiode reading. Then adjust the position of the slit-blocker to let the light to pass through only one of the slits, and measure the change in the photodiode signal.

- To see another and even more dramatic manifestation of the wave nature of light, set the slit blocker again to permit light from both slits to pass along the apparatus, and now place the detector slit at either of the minima immediately adjacent to the central maximum; take some care to find the very bottom of this minimum. Record what happens when you use the slit-blocker to block the light from one, or the other, of the two slits?

- Check your experimental results against the theoretical predictions using Eqs. (1.1) and (1.2). Do your observation confirm or contradict wave theory?

Once you have performed these spot-checks, and have understood the motivation for them and the obtained results, you are ready to conduct systematic measurements of intensity distribution (the photodiode voltage-output signal) as a function of detector slit position. You will make such **measurements in two slit-blocker positions: when both slits are open, and when only one slit is open**. You will need to take enough data points to reproduce the intensity distribution in each case. Taking points systematically every 0.05 or 0.1 mm on the tick lines will produce a very high quality dataset. One person should turn the dial and the other should record readings directly to paper or a spreadsheet (if you do this, print it out and tape into your logbook). Estimate your uncertainties from the dial and the voltmeter. Cycle through multiple maxima and minima on both sides of the central maximum. It is a good idea to plot the data points immediately along with the data taking – nothing beats an emerging graph for teaching you what is going on, and your graph will be pretty impressive. *Note: due to large number of points you don't need to include the tables with these measurements in the lab report – the plotted distributions should be sufficient. Be clear on your uncertainties though.*

Slit separation calculations: Once you have enough data points for each graph to clearly see the interference pattern, use your data to extract the information about the distance between two slits d. To do that find the positions of consecutive interference maxima or minima, and calculate average d using Eq. 1.2. Estimate the uncertainty in these parameters due to laser wavelength uncertainty. Check if your measured values are within experimental uncertainty from the manufacturer's specs: the center-to-center slit separation is 0.353 mm (or 0.406 or 0.457 mm, depending on what two-slit mask you have installed).

Fit your data with Eqs. (1.1) and (1.2). You will need to add these functions using "Add new function" option. Note that in this case you will have to provide a list of initial guesses for all the fitting parameters. A few tips:

- Make sure that units of all your measured values are self-consistent - the program will go crazy trying to combine measurements in meters and micrometers together!

- Try to plot your function for guesstimated values before doing a fit with it. This catches many silly errors.

- You will have to fit for a term to account for the overall normalization and also for the fact that the maximum is not set at $x = 0$. In other words, substitute $x \to x - x_0$.

Estimate both of these (for the normalization, look at what happens when $x - x_0 = 0$) but include them as free parameters in the fit.

- You need to add a parameter to account for the non-zero background you observed when the laser was off.
- Include the minimal number of parameters. If I have parameters a and b but they always appear as ab in my function, then I am much better off including a term $c = ab$ when doing the fit. Otherwise the fit is underconstrained. Adjusting a up has the same effect as adjusting b down so it's impossible to converge on unique values for a and b. Fitting algorithms really dislike this situation.
- If the program has problems fitting all the parameters, first hold the values of the parameters you know fairly well (such as the light wavelength, maximum peak intensity, background, etc.). Once you determine the approximate values for all other parameters, you can release the fixed ones, and let the program adjust everything to make the fit better.

Single-photon interference

Before you start the measurements you have to convince yourself that the rate of photons emitted by the weak filtered light bulb is low enough to have in average less than one photon detected in the apparatus at any time. Roughly estimate the number of photons per second arriving to the detector. First, calculate the number of photons emitted by the light bulb in a 10 nm spectral window of the green filter (between 541 and 551 nm), if it runs at 6V and 0.2A, only 5% of its electric energy turns into light, and this optical energy is evenly distributed in the spectral range between 500 nm and 1500 nm. These photons are emitted in all directions, but all of then are absorbed inside the box except for those passing through two slits with area approximately 0.1×10 mm^2. Next, if we assume that the beam of photons passing through the slits diffract over a 1 cm^2 area by the time they reach the detector slit, estimate the rate of photons reaching the detector. Finally, we have to adjust the detected photon rate by taking into account that for PMT only 4% of photons produce output electric pulse at the output. That's the rate of event you expect. Now estimate the time it takes a photon to travel through the apparatus, and estimate the average number of detectable photons inside at a given moment of time. *You may do this calculations before or after the lab period, but make sure to include them in the lab report.*

Now you need to change the apparatus to use the light bulb. Open the cover and slide the laser source to the side (do not remove the laser from the stand). Now set the 3-position toggle switch to the BULB position and dial the bulb adjustment up from 0 until you see the bulb light up. (*The flashlight bulb you're using will live longest if you minimize the time you spend with it dialed above 6 on its scale, and if you toggle its power switch only when the dial is set to low values*). If the apparatus has been aligned, the bulb should now be in position to send light through the apparatus. Check that the green filter-holding structure is in place: the light-bulb should look green, since the green filter blocks nearly all the light

emerging from the bulb, passing only wavelengths in the range 541 to 551 nm. The filtered light bulb is very dim, and you probably will not be able to see much light at the double slit position even with room light turned off completely. No matter; plenty of green-light photons will still be reaching the double-slit structure – in fact, you should now dim the bulb even more, by setting its intensity control down to about 3 on its dial.

Now close and lock the cover - you are ready to start counting photons. But first a WARNING: a photomultiplier tube is so sensitive a device that it should not be exposed even to moderate levels of light when turned off, and must not be exposed to anything but the dimmest of lights when turned on. In this context, ordinary room light is intolerably bright even to a PMT turned off, and light as dim as moonlight is much too bright for a PMT turned on.

Direct observation of photomultiplier pulses You will use a digital oscilloscope for first examination of the PMT output pulses, and a digital counter for counting the photon events. Set the oscilloscope level to about 50 mV/division vertically, and 250 - 500 ns/division horizontally, and set it to trigger on positive-going pulses or edges of perhaps > 20 mV height. Now find the PHOTOMULTIPLIER OUTPUT of the detector box, and connect it via a BNC cable to the vertical input of the oscilloscope. Keeping the shutter closed, set the HIGH-VOLTAGE 10-turn dial to 0.00, and turn on the HIGH-VOLTAGE toggle switch. Start to increase the voltage while watching the scope display. *If you see some sinusoidal modulation of a few mV amplitude, and of about 200 kHz frequency, in the baseline of the PMT signal, this is normal. If you see a continuing high rate (> 10 kHz) of pulses from the PMT, this is not normal, and you should turn down, or off, the bias level and start fresh – you may have a malfunction, or a light leak.* Somewhere around a setting of 4 or 5 turns of the dial, you should get occasional positive-going pulses on the scope, occurring at a modest rate of $1 - 10$ per second. If you see this low rate of pulses, you have discovered the "dark rate" of the PMT, its output pulse rate even in the total absence of light. You also now have the PMT ready to look at photons from your two-slit apparatus, so finally you may open the shutter. The oscilloscope should now show a much greater rate of pulses, perhaps of order 10^3 per second, and that rate should vary systematically with the setting of the bulb intensity. *You may find a small device called Cricket in your table. It allows you to "hear" the individual photon arrivals - ask your instructor to show you how it works.*

To count the pulses using a counter you will use another PMT output – the OUTPUT TTL – that generates a single pulse, of fixed height and duration, each time the analog pulse exceeds an adjustable threshold. To adjust the TTL settings display the OUTPUT TTL on the second oscilloscope channel and set it for 2 V/div vertically. By simultaneously watching both analog and TTL-level pulses on the display, you should be able to find a discriminator setting, low on the dial, for which the scope shows one TTL pulse for each of, and for only, those analog pulses which reach (say) a +50 -mV level. If your analog pulses are mostly not this high, you can raise the PMT bias by half a turn (50 Volts) to gain more electron multiplication. If your TTL pulses come much more frequently than the analog pulses, set the discriminator dial lower on its scale.

Now send the TTL pulses to a counter, arranged to display successive readings of the

number of TTL pulses that occur in successive 1-second time intervals. To confirm that this is true, record a series of "dark counts" obtained with the light bulb dialed all the way down to 0 on its scale. Now choose a setting that gives an adequate photon count rate (about 10^3/second) and use the slit-blocker, according to your previously obtained settings, to block the light from both slits. This should reduce the count rate to a background rate, probably somewhat higher than the dark rate. Next, open up both slits, and try moving the detector slit to see if you can see interference fringes in the photon count rate. You will need to pick a detector-slit location, wait for a second or more, then read the photon count in one or more 1-second intervals before trying a new detector-slit location. If you can see maxima and minima, you are ready to take data. Finally, park the slit near the central maximum and choose the PMT bias at around 5 turns of the dial and the bulb intensity setting to yield some convenient count rate ($10^3 - 10^4$ events/second) at the central maximum.

Single-photon detection of the interference pattern. Most likely the experimental results in the previous section has demonstrated good agreement with the wave description of light. However, the PMT detects individual photons, so one can expect that now one has to describe the light beam as a stream of particles, and the wave theory is not valid anymore. To check this assumption, you will repeat the measurements and take the same sort of data as in the previous section, except now characterizing the light intensity as photon count rate.

- Like previously, slowly change the position of the detection slit and record the average count rate in each point. Start with the two-slit interference. Plot the data and confirm that you see interference fringes.

- Repeat the measurement with one slit blocked and make the plot.

- Use the spacing of the interference maxima to check that the light source has a different wavelength than the red laser light you used previously. Using the previously determined value of the slit separation d, calculate the wavelength of the light, and check that it is consistent with the green filter specs ($541 - 551$ nm).

The plots of your experimental data are clear evidence of particle-wave duality for photons. You've made contact with the central question of quantum mechanics: how can light, which so clearly propagates as a wave that we can measure its wavelength, also be detected as individual photon events? Or alternatively, how can individual photons in flight through this apparatus nevertheless 'know' whether one, or both, slits are open, in the sense of giving photon arrival rates which decrease when a second slit is opened? Discuss these issues in your lab report.

Two-Slit interference with atoms[1]

According to quantum mechanics, the wave-particle duality must be applied not only to light, but to any "real" particles as well. That means that under the right circumstance, atoms

[1]Special thanks to Prof. Seth Aubin for providing the materials for this section

should behave as waves with wavelength $\lambda_{\text{atom}} = h/\sqrt{2mE} = h/p$ *(often called deBroglie wavelength), where* h *is Planck's constant,* m *is the mass of the particle, and* E *and* p *are respectively the kinetic energy and the momentum of the particle. In general, wave effects with "massive" particles are much harder to observe compare to massless photons, since their wavelengths are much shorter. Nevertheless, it is possible, especially now when scientists has mastered the tools to produce ultra-cold atomic samples at nanoKelvin temperatures. As the energy of a cooled atom decreases, its deBroglie wavelength becomes larger, and the atom behaves more and more like waves. For example, in several experiments, researches used a Bose-Einstein condensate (BEC) – the atomic equivalent of a laser – to demonstrate the atomic equivalent of the Young's double slit experiment. As shown in Fig. 1.3(a), an original BEC sits in single-well trapping potential, which is slowly deformed into a double-well trapping potential thus producing two phase-coherent atom wave sources. When the trapping potential is turned off, the two BECs expand and interfere where they overlap, just as in the original Young's double slit experiment.*

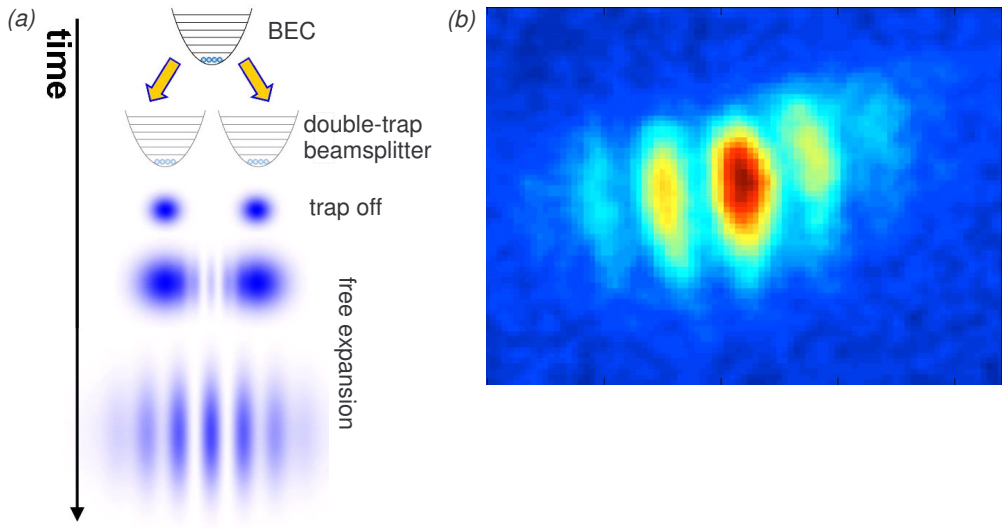

Figure 1.3: Atom interferometry version of Young's double slit experiment: *(a)* schematic and *(b)* experimentally measured interference pattern in an ^{87}Rb Bose-Einstein condensate.

Fig. 1.3(b), shows the resulting interference pattern for a ^{87}*Rb BEC. Atom interferometry is an area of active research, since atoms hold promise to significantly improve interferometric resolution due their much shorter deBroglie wavelength compared to optical photons. In fact, the present most accurate measurements of accelerations, rotations, and gravity gradients are based on atomic interference.*

Appendix: Fraunhofer Diffraction at a Single Slit and Two-Slit interference

Diffraction at a Single Slit We will use a *Fraunhofer* diffraction model to calculate the intensity distribution resulting from light passing a single slit of width a, as shown in Fig. 1.4(a). We will assume that the screen is far away from the slit, so that the light beams passed through different parts of the slit are nearly parallel. To calculate the total intensity on the

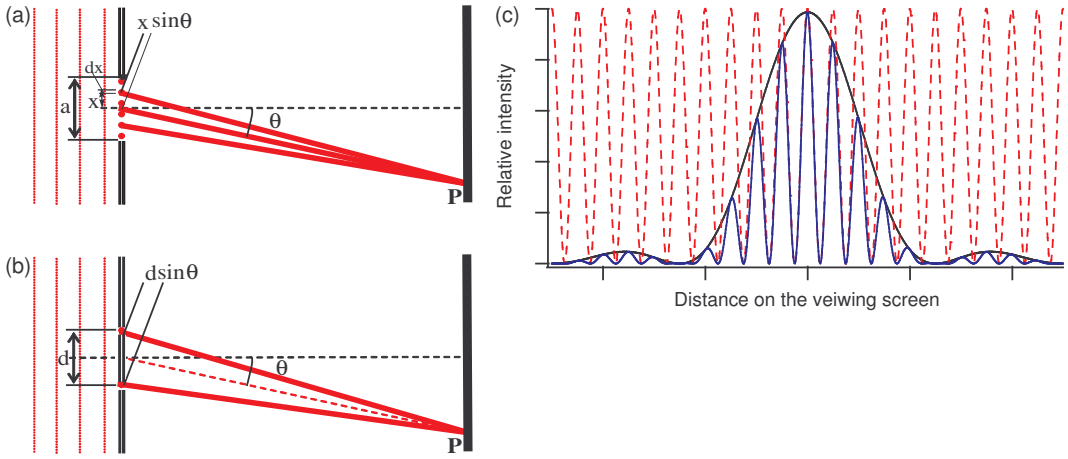

Figure 1.4: (a) Single slit diffraction pattern formation. (b) Two-slit interference pattern formation. (c) Examples of the intensity distributions on a viewing screen after passing one slit (black), two infinitely small slits (red), two slits of finite width (blue).

screen we need to sum the contributions from different parts of the slit, taking into account phase difference acquired by light waves that traveled different distance to the screen. If this phase difference is large enough we will see an interference pattern. Let's break the total height of the slit by very large number of point-like radiators with length dx each and positioned at the height x above the center of the slit (see Fig. 1.4(a)). Since it is more convenient to work with complex numbers, we will assume that the original incident wave is a real part of $E(z,t) = E_0 e^{ikz - i2\pi\nu t}$, where $k = 2\pi/\lambda$ is the wave number. Then the amplitude of each point radiator on a slit is [a real part of] $dE(z,t) = E_0 e^{ikz - i2\pi\nu t} dx$. A beam emitted by a radiator at the height x above the center of the slit must travel an extra distance $x \sin\theta$ to reach the plane of the screen, acquiring an additional phase factor. Then we may write a contributions at the point P from a point radiator dx as the real part of:

$$dE_P(z,t,x) = E_0 e^{ikz - i2\pi\nu t} e^{ikx\sin\theta} dx. \tag{1.3}$$

To find the overall amplitude at that point we need to add up the contributions from all point sources along the slit:

$$E_P = \int_{-a/2}^{a/2} dE(z,t) = E_0 e^{ikz - i2\pi\nu t} \int_{-a/2}^{a/2} e^{ikx\sin\theta} dx = A_P \times E_0 e^{ikz - i2\pi\nu t}. \tag{1.4}$$

Here A_P is the relative amplitude of the electromagnetic field at the point P:

$$A_P = \frac{1}{ik\sin\theta}\cdot\left(e^{ik\frac{a}{2}\sin\theta} - e^{-ik\frac{a}{2}\sin\theta}\right) \propto \frac{\sin(\frac{\pi D}{\lambda}\sin\theta)}{\frac{\pi D}{\lambda}\sin\theta} \tag{1.5}$$

The intensity is proportional to the square of the amplitude and thus

$$I_P \propto \frac{(\sin(\frac{\pi a}{\lambda}\sin\theta))^2}{(\frac{\pi a}{\lambda}\sin\theta)^2} \tag{1.6}$$

The minima of the intensity ("dark fringes") occur at the zeros of the argument of the sin function: $\frac{\pi D}{\lambda}\sin\theta = m\pi$, while the maxima ("bright fringes") are almost exactly match $\frac{\pi D}{\lambda}\sin\theta = (m+\frac{1}{2})\pi$ for $m = 0, \pm1, \pm2, \cdots$.

Let us now consider the case of interference pattern from two identical slits separated by the distance d, as shown in Fig. 1.4(b). We will assume that the size of the slits is much smaller than the distance between them, so that the effect of Fraunhofer diffraction on each individual slit is negligible. Then going through the similar steps the resulting intensity distribution on the screen is given my familiar Young formula:

$$I(\theta) = \left|E_0 e^{ikd/2\sin\theta} + E_0 e^{-ikd/2\sin\theta}\right|^2 = 4I_0\cos^2\left(\frac{\pi h}{\lambda}\sin\theta\right), \tag{1.7}$$

where $k = 2\pi/\lambda$, $I_0 = |E_0|^2$, and the angle θ is measured with respect to the normal to the plane containing the slits.

If we now include the Fraunhofer diffraction on each slit as we did before, we arrive to the total intensity distribution for two-slit interference pattern:

$$I(\theta) \propto \cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right)\left[\frac{\sin(\frac{\pi a}{\lambda}\sin\theta)}{\frac{\pi a}{\lambda}\sin\theta}\right]^2. \tag{1.8}$$

The examples of the light intensity distributions for all three situations are shown in Fig. 1.4(c). Note that the intensity distributions derived here are functions of the angle θ between the normal to the plane containing the slits and the direction to the point on the screen. To connect these equations to Eqs. (1.1) and(1.2) we assume that $\sin\theta \simeq \tan\theta = x/\ell$ where x is the distance to the point P on the screen, and ℓ is the distance from the two slit plane to the screen.