Interlayer Transport in Bilayer Quantum Hall Systems

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Bilayer quantum Hall systems have a broken symmetry ground state at a filling factor $\nu = 1$ which can be viewed either as an excitonic superfluid or as a pseudospin ferromagnet. We present a theory of interlayer transport in quantum Hall bilayers that highlights remarkable similarities and critical differences between transport in Josephson junction and ferromagnetic metal spin-transfer devices. Our theory is able to explain the size of the large but finite low-bias interlayer conductance and the voltage width of this collective transport anomaly.

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Introduction.—Quantum Hall bilayers at a total Landau level filling factor $\nu = 1$ have ground states with spontaneous interlayer phase coherence [1]. These broken symmetry states can be regarded equivalently either as pseudospin ferromagnets or as excitonic superfluids [2]. One of the most spectacular experimental manifestations of this order is an enormous low-temperature enhancement [3] of the interlayer tunneling conductance in samples with extremely small interlayer tunneling amplitudes. The differential tunnel conductance is large only at small bias voltages and reaches a finite maximum value that can be as large as $\sim 0.5e^2/h$ at low temperatures. Although these important observations [3,4] have inspired considerable interesting theoretical activity [5], it has not been possible to account for their main qualitative features, in particular, for the voltage width of the anomaly, the finite value of the conductance maximum, and the inverse relationship between these two quantities. In this Letter we present a theory of the low-bias tunneling anomaly which, in contrast to most previous theoretical work, predicts that the zero-bias conductance is finite even in a perfect disorder free bilayer at temperature $T = 0$, and accounts approximately for the width of the anomaly. Our theory sees interlayer tunneling phenomena as partially analogous to both tunneling across a Josephson junction [6] and the spin-transfer phenomena in ferromagnetic metals [7–9]. The key difference between these two examples of current driven order parameter manipulation is that the bias is applied by a superconducting condensate in the former case and by dissipative quasiparticles in the latter. Tunneling in quantum Hall bilayers is an example of pseudospin transfer, with the feature particular to quantum Hall systems that transport quasiparticles are localized at the edge of the system when order is strong and the quantum Hall effect firmly established.

Josephson junction and spin-transfer phenomenology.—The semiclassical equations of motion for the collective dynamics of a current-biased Josephson Junction and a single-domain easy-plane ferromagnet in an in-plane field are similar:

$$\begin{aligned} \hbar\dot{\phi} &= 2\ \text{eV} + \alpha_\phi \dot{N}, \\ \dot{N} &= -\frac{I_c}{2e}\sin(\phi) - \alpha_G\dot{\phi} + \frac{I_{\text{bias}}}{2e}, \end{aligned} \tag{1}$$

and

$$\begin{aligned} \hbar\dot{\phi} &= \frac{K_z}{M_0}M_z + \alpha_\phi \dot{M}_z, \\ \dot{M}_z &= -\frac{H_x}{g\mu_B\hbar}M_0\sin(\phi) - \alpha_\theta M_0\dot{\phi} + \frac{g_{\text{ST}}I}{e}. \end{aligned} \tag{2}$$

In Eq. (1) V is the voltage difference across the junction, produced partly by the external circuit and partly capacitively by junction charging; N is the number of Cooper pairs accumulated on one side of the junction, α_G is a dissipation coefficient due to thermally activated quasiparticle conductance and coupling to the external circuit; and α_ϕ is an additional dissipation coefficient that is normally negligible (and often neglected). I_{bias} is the bias current which represents the influence of the rest of the superconducting circuit on the junction region. The dc Josephson effect occurs when the condition for quasiparticles to be in equilibrium with the condensate, $I_c\sin(\phi) = I_{\text{bias}}$, is satisfied. The current bias influences the microscopic self-energy of the quasiparticles so that the phase change across the junction ϕ is no longer zero when the quasiparticle density matrix has its equilibrium value. Current then flows across the junction without dissipation.

In Eq. (2), α_ϕ and α_θ are the dissipation coefficients that appear in the Landau-Lifshitz-Gilbert (LLG) equations for ferromagnets, K_z is the magnetic anisotropy coefficient which is normally dominated in thin film magnets by magnetostatic interactions (shape anisotropy), and $g_{\text{ST}}I/e$ is the spin-transfer torque. The presence of this current bias term in the collective magnetization equations of motion [7,9] can be inferred from the approximate conservation of total spin in typical itinerant electron ferromagnets. The version of the spin-torque LLG equations specified by Eq. (2), applies when the transport current incident on the magnetic nanoparticle of interest has a spin orientation perpendicular to the easy plane, the circumstance relevant to tunneling in bilayer quantum Hall systems as we see below. g_{ST} is the spin-transfer efficiency factor which is typically ~ 1 and depends on both the degree of polarization of the injected current and the rest of the circuit. The spin-transfer torque term in the equation of motion, like the bias term in the Josephson junction case, arises [10] microscopically from a change in quasiparticle self-energy; in the presence of the transport current, the quasiparticles are in equilibrium not when the in-plane magnetization is aligned with the LLG equation effective magnetic field, but when its orientation in the easy-plane is displaced from the field direction by an angle proportional to the current. The key difference between a current-biased Josephson junction and a current-biased nanomagnet is that the bias field experienced by the quasiparticles is applied in one case by the condensate, and in the other case by transport quasiparticles that are held out of equilibrium by a finite bias voltage. The current in spin-transport devices is always dissipative. Unlike a Josephson junction, a finite voltage drop occurs across a nanomagnet when a spin-current bias is applied.

Interlayer transport in quantum Hall bilayers.—We now discuss interlayer tunneling in bilayer quantum Hall systems, starting with the ideal disorder free case. We use a pseudospin ferromagnet language in which electrons in the top layer have pseudospin up and electrons in the bottom layer have pseudospin down. In the ordered state, quasiparticles in the bulk of a quantum Hall bilayer have an interlayer tunneling amplitude that includes a self-energy [1,2,11] contribution which can be represented by an in-plane pseudospin effective magnetic field:

$$\Delta_x(X) = \Delta_t + \frac{1}{L}\sum_{X'} n_{X'} F_D(X - X')\cos(\phi_{X'}),$$
$$\Delta_y(X) = \frac{1}{L}\sum_{X'} n_{X'} F_D(X - X')\sin(\phi_{X'}). \quad (3)$$

In Eq. (3), Δ_t is the single-particle tunneling amplitude which for the systems in question has an extremely small [3] value $\sim 10^{-8}$ eV, $X = l^2 k$ labels a guiding center state which is delocalized along the edge of the system (see Fig. 1), $n_{X'}$ is a guiding center state occupation number, L is the length of the system along the edge, $\phi_{X'}$ is the planar pseudospin orientation (or, equivalently, the phase difference between top and bottom layers) for guiding center X' and $F_D(X - X')$ is [11] the interlayer exchange integral between guiding centers X and X'. In the absence of a transport current, the quasiparticle pseudospin will align with the effective magnetic field for each guiding center:

$$\frac{\cos(\phi_X)}{\sin(\phi_X)} = \frac{\Delta_x(X)}{\Delta_y(X)}. \quad (4)$$

It is easy to verify that in this case the only self-consistent solution to Eqs. (3) and (4) is $\sin(\phi_X) \equiv 0$; the small single-particle tunneling amplitude selects the phase difference between the two layers and the effective tunneling amplitude is enormously enhanced by interactions $[\Delta_t \to \Delta_x(X)]$.

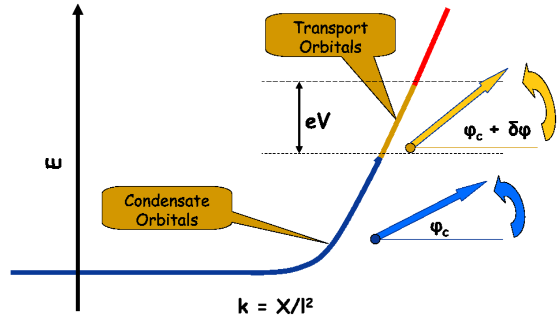

FIG. 1 (color online). Schematic illustration of pseudospin transfer torques in quantum Hall bilayers. The transport orbitals on a quantum Hall plateau are extended along the edge of the system. In the absence of disorder, wave vector k is a good quantum number and is proportional to the cyclotron-orbit guiding center $X = kl^2$. Each orbital makes a contribution to the pseudospin exchange field that is in the direction of its pseudospin orientation. Unlike equilibrium orbitals, transport orbitals do not align with their pseudospin fields, giving rise to a pseudospin transfer torque. Large interlayer conductance occurs when the pseudospin transfer torque can be canceled by a pseudospin torque due to the bare interlayer tunneling term in the Hamiltonian.

When the system carries a current, the quasiparticle's Schrödinger equation must be solved with the scattering boundary condition that the current is incident from the high chemical potential contact. Under these circumstances, its pseudospin orientation will *not* align [10] with its effective magnetic field. Since all electrons move along the edge at the magnetoplasmon velocity [12] $v_{\rm emp} \sim 2 \times 10^6$ m s^{-1}, the quasiparticle Schrödinger equation may be mapped to that of a zero-dimensional spin $S = 1/2$ particle in a time-dependent field. Differences between the disorder potentials in the two layers will give rise to a pseudospin effective field in the $\hat{z}$ direction which varies randomly along the edge. Averaging along the edge, the rate of spin precession from up (top layer) to down (bottom layer) is proportional to the mean torque that acts on the planar spin. We find that for the transport electrons

$$\frac{\Delta_{\rm QP}^E}{\hbar}\sin(\phi_{\rm tr} - \phi_c) = \frac{g}{t_{\rm transit}} = \frac{g v_{\rm emp}}{L}. \quad (5)$$

In Eq. (5), $\phi_c = \tan^{-1}(\Delta_y/\Delta_x)$ is the orientation of the pseudospin effective field at the edge, $\Delta_{\rm QP}^E$ is the magnitude of the exchange effective magnetic field at the edge of the system $[\Delta_{\rm QP}^E = L^{-1}\sum_{X'<X} F_D(X - X')$ up to negligible corrections from Δ_t and is half the bulk quasiparticle gap], g is the probability that an electron injected in the top layer will make its way to the bottom layer, and $t_{\rm transit} = L/v_{\rm emp}$ is the time required for an edge electron to transit the sample. Equation (5) is most easily under-

stood in a reference frame that moves along the edge (at the magnetoplasmon velocity) with the transport electrons; from this point of view it merely asserts that a spin torque due to misalignment between the exchange field and the in-plane pseudospin orientation is necessary to produce the required precession from top layer (pseudospin up) to bottom layer (pseudospin down). We refer to ϕ_c below as the condensate pseudospin orientation. Since $g < 1$, $\Delta_{\rm QP} \sim 10^{-4}$ eV [11] and $L \sim 10^{-2}$ cm in typical samples, it follows that $\delta\phi \equiv \phi_{\rm tr} - \phi_c$ is small and therefore that $\sin(\delta\phi) \approx \delta\phi$.

We propose that the tunneling anomaly in quantum Hall bilayers occurs when it is possible to achieve a self-consistent solution of the mean-field equations in the presence of current induced pseudospin torques. Combining Eqs. (3) and (5) we obtain

$$\Delta_x(X_e) = \Delta_t + \Delta^E_{\rm QP}\cos(\phi_c) + \frac{F_D(0)N_{\rm tr}}{L}\cos(\phi_c + \delta\phi),$$

$$\Delta_y(X_e) = \Delta^E_{\rm QP}\sin(\phi_c) + \frac{F_D(0)N_{\rm tr}}{L}\sin(\phi_c + \delta\phi). \qquad (6)$$

In this equation $N_{\rm tr} = L\ {\rm eV}/(hv_{\rm emp})$ is the number of edge states in the narrow transport window with energy width eV. (See Fig. 1.) The bias voltage is assumed to be small enough that the guiding center width of the transport window is much smaller than the range [11] ($\sim l$ where l is the magnetic length) of the $F_D(X - X')$ exchange integral. The terms proportional to $\Delta^E_{\rm QP}$ in Eq. (6) are edge self-energies in the absence of transport currents and the terms proportional to $F_D(0)$ are the pseudospin torque contributions. Equation (6) neglect the variation of ϕ_c from its maximally deflected value at the edge to the its value deep in the bulk ($\phi_c = 0$) since these occur on a length scale ($l\sqrt{\Delta^E_{\rm QP}/\Delta_t}$), much longer than the range of $F_D(X - X')$. Solving Eq. (6) we find that

$$\Delta_t \sin(\phi_c) = \frac{F_D(0)N_{\rm tr}}{L}\delta\phi \qquad (7)$$

and hence, that the maximum bias voltage for which a time-independent solution of the mean-field equations exists is

$$e\,V^* = \frac{\Delta_t v_{\rm emp} h}{F_D(0)\delta\phi} = \frac{2\pi\Delta_t\Delta^E_{\rm QP}L}{gF_D(0)}. \qquad (8)$$

Since [11] $F_D(0) \sim e^2/(\epsilon) \sim 10^{-2}$ eVl, it follows that the width of the low-bias tunneling anomaly when the quantum Hall effect is most strongly developed (at the lowest temperatures) should be $\sim 10^{-6}$ eV, consistent with experiment.

Landau-Zener tunneling.—Having established our basic picture, we now comment on the surprisingly rapid decrease of the interlayer conductance with increasing temperature, and on the extreme sample quality required to approach the highest zero-bias conductance values. When a time-independent solution of the mean-field equations is possible, the quasiparticle conductance can be evaluated using the Landauer-Buttiker [13] scattering theory picture of transport which predicts conductance ge^2/h. The maximum possible value $g = 1/2$ applies when an electron injected in one layer has a 50-50 chance of being found in either layer at later times. In the absence of disorder, g should approach 0.5 when the pseudospin precession length $L_{\rm pr} \equiv v_{\rm emp}\hbar/\Delta^E_{\rm QP}$ is less than L, i.e., when $\Delta^E_{\rm QP}$ is larger than $\sim 10^{-5}$ eV, a condition that we expect to be satisfied quickly once the ordered state is entered. Instead, g is almost always substantially smaller than 1 in experiment. Interlayer tunneling that is strongly suppressed compared to naive expectations has been seen [14,15] previously in bilayers with large purely single-particle tunneling amplitudes. In both cases, we ascribe the behavior to the combined effects of disorder and large edge magnetoplasmon velocities.

As illustrated in Fig. 2 electrons traveling along the edge at velocity $v_{\rm emp}$ see differences between the random disorder potentials in the two layers as $\hat{z}$ direction pseudospin magnetic fields. The typical rate at which the pseudospin field varies is $v_{\rm emp}V_{\rm dis}/L_{\rm dis}$ where $V_{\rm dis}$ and $L_{\rm dis}$ are the typical size and correlation length of the potential difference between the layers. (In the bulk of the two-dimensional electron system these random pseudospin field fluctuations are screened [1,11] by tilting the pseudospin slightly out of the easy plane.) From floating gate screened disorder potential measurements in the quantum Hall regime [16] it follows that $L_{\rm dis} \sim 10^{-6}$ m so that $\sim L/L_{\rm dis} \sim 100$. Avoided crossings of adiabatic pseudospin energy levels occur when the disorder potential difference changes sign. Quasiparticles that follow the adiabatic evolution path will transfer between layers at

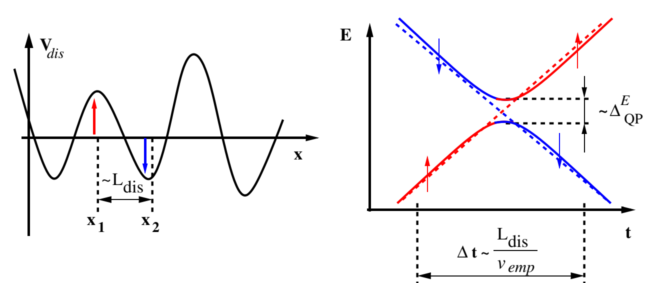

FIG. 2 (color online). Schematic illustration of the suppression of interlayer tunneling by disorder. Differences between the random potentials in the two layers give rise to substantial pseudospin fields in the $\hat{z}$ direction, $V_{\rm dis}$, which are not effectively screened at the edge of the system. Along much of the edge, $V_{\rm dis}$ is larger than the in-plane coherence induced exchange field $\Delta^E_{\rm QP}$. When the disorder potential difference changes sign, quasiparticles will cross between layers if they follow the adiabatic path. Because of the high velocity of edge excitations, quasiparticles are likely to Landau-Zener tunnel to the higher energy state and remain in the same layer.

each avoided crossing. We estimate g as the product of the number of avoided crossings (disorder potential difference sign changes) and the probability of transferring between layers at an individual crossing [19]:

$$g = (L/L_{\rm dis})(1 - \exp[-2(\Delta_{\rm QP}^E)^2 L_{\rm dis}/\hbar v_{\rm emp} V_{\rm dis}]) \sim 2(\Delta_{\rm QP}^E)^2 L/\hbar v_{\rm emp} V_{\rm dis}. \quad (9)$$

The strength of the bare disorder potential $V_{\rm dis}$ in typical high-mobility samples can be estimated by floating gate measurements [16]; we find that $V_{\rm dis}$ is comparable to the zero-field Fermi energy $\sim 10^{-3}$ eV. It follows that the argument of the exponential function in the Landau-Zener tunneling formula is small even for the $T = 0$ values of $\Delta_{\rm QP}^E$, consistent with experiment. This rough estimate of the typical value of $V_{\rm dis}$ is consistent with the observations that g approaches $1/2$ only at the lowest temperatures and justifies the small argument expansion used in the final form for the right-hand side of Eq. (9).

Equation (5) can be applied consistently with Eq. (9) because of the long range of the Coulombic edge exchange interaction which can average over a number of potential interlayer tunneling sites. Combining Eqs. (9) and (8) we find that

$$\mathrm{e\,V}^* = \frac{\Delta_t}{\Delta_{\rm QP}^E} \frac{h V_{\rm dis} v_{\rm emp}}{F_D(0)}. \quad (10)$$

Equation (9) for the zero-bias interlayer tunnel conductance, and Eq. (10) for the voltage width of the low-bias anomaly, are the main predictions of this Letter. In our theory, the temperature dependence of the transport anomaly follows from thermal fluctuations in the condensate phase which reduce the order parameter and $\Delta_{\rm QP}^E$ [11,17]. The increase [3] of eV^* by a factor of approximately 40 between 20 mK and 0.3 K is consistent with the size of suppression that is expected, although the detailed behavior is certainly disorder dependent and sample specific. The decrease [3] in zero-bias conductance by a factor of 2000 over the same temperature interval is then consistent with the predictions of Eq. (9). Our theory also accounts qualitatively for in-plane field $B_{\|}$ dependence of the anomaly which is marked [3] by a strong decrease in conductance with little change in voltage width. This behavior is predicted by our theory since $\Delta_{\rm QP}^E$ and Δ_t have similar field dependence, both dropping [11,18] by a factor ~ 1 when $B_{\|}/B_{\rm perp} \sim d/l$.

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