Chiral Superfluid States in Hybrid Graphene Heterostructures

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We study the "hybrid" heterostructure formed by one sheet of single-layer graphene (SLG) and one sheet of bilayer graphene (BLG) separated by a thin film of dielectric material. In general, it is expected that interlayer interactions can drive the system to a spontaneously broken-symmetry state characterized by interlayer phase coherence. The peculiarity of the SLG-BLG heterostructure is that the electrons in the two layers have different chiralities. We find that this difference causes the spontaneously broken-symmetry state to be N-fold degenerate. Moreover, we find that some of the degenerate states are chiral superfluid states, topologically distinct from the usual layer ferromagnetism. The chiral nature of the ground state opens the possibility to realize protected midgap states. The N-fold degeneracy of the ground state makes the physics of SLG-BLG hybrid systems analogous to the physics of ^{3}He, in particular given the recent discovery of chiral superfluid states in this system.

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Graphene [1] and bilayer graphene [2] are ideal two-dimensional electronic systems [3,4] in which the conduction and valence bands touch at single points, charge neutrality points, at the corners of the Brillouin zone (BZ). Around these points, the low-energy electronic states are well described as massless Dirac fermions with Berry phase π in single layer graphene (SLG) and as massive chiral fermions with Berry phase 2π in bilayer graphene (BLG). Recently, the use of hexagonal boron nitride (BN) films [5] has allowed the realization of graphene heterostructures [6,7] in which the graphene layers are only few nanometers apart and still electrically isolated [8–12]. In this situation, interlayer interactions can drive the system into an interlayer phase coherent ground state [13–16]. This state can be thought of as an exciton condensate [17,18] of electrons in one layer and holes in the other layer, as a superfluid state [19], or by treating the layer degree of freedom as a spin degree of freedom (pseudospin) as a ferromagnetic state. Experimental evidence suggests that the interlayer phase coherent state has been realized in quantum Hall bilayers [20–26] and very recently [12] in symmetric double-layer graphene systems. The experimental capability to realize high-quality graphene-BN heterostructures has made it possible to study the effects of interactions between fermionic quasiparticles having qualitatively different dispersion and chirality. This can be realized by creating heterostructures in which one layer is SLG and the other is BLG.

In this Letter, we study the nature of the interlayer broken-symmetry state for SLG-BN-BLG systems. We find that the difference in the dispersion and chirality between the two layers profoundly modifies the nature of the ground state. In particular, we find that due to the difference of chirality (i) the interlayer broken-symmetry state is N-fold degenerate ($N = 2$ or 4 depending on the nature, long-range or short-range, of the interlayer interaction) and (ii) one of the degenerate states is always chiral, i.e., characterized by a complex order parameter whose phase depends on the momentum direction. The N-fold degeneracy of the ground state raises the possibility that in SLG-BLG systems a state could be realized analogous to states realized in ^{3}He [27,28]. Moreover, the chiral nature of one of the ground states makes possible the realization of protected midgap states in the presence of vortices in the exciton condensate [29–31].

The heterostructure that we study is shown schematically in Fig. 1. The two layers are connected to separate gates (V_g^S, $-V_g^B$) so that their doping can be controlled independently and can be adjusted to have the p-type Fermi surface (FS) in one layer nested with the n-type FS in the other, a condition that favors the instability toward the formation of the exciton condensate. Let $V_g = V_g^B + V_g^S$ be the bias voltage for which the FSs in

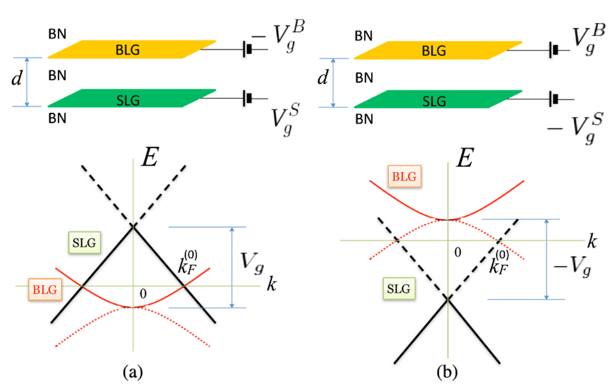

FIG. 1 (color online). (a) SLG and BLG are gated individually at voltages V_g^S and $-V_g^B$, $V_g = V_g^S + V_g^B$. At low energies and low voltages, the most relevant bands are the BLG conduction band and the SLG valence band. (b) By inversion of the voltages ($-V_g^S$ and V_g^B), the most relevant bands become the SLG conduction band and the BLG valence band.

BLG and SLG are nested. At low energies, the band structure of SLG is well described by two inequivalent valleys (at the K and K' points in the BZ) around which the fermionic dispersion is linear. In BLG, the low-energy conduction and valence bands also touch at the points K K', but around these points the dispersion is nearly parabolic with an effective mass $m \approx 0.03 m_e$ [3,4]. For this experimental setup, the effective low-energy band structure is formed by the conduction band of BLG and the valence band of SLG [or vice versa as shown in Fig. 1(b)].

The low-energy physics of the SLG-BLG system is described by the Hamiltonian $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\rm int}$, where, in the limit of vanishing interlayer tunneling, the noninteracting Hamiltonian $\mathcal{H}_0 = \sum_{\mathbf{k},\sigma} \varepsilon_{\mathbf{k},\sigma} c^\dagger_{\mathbf{k},\sigma} c_{\mathbf{k},\sigma}$ with $\sigma = 1$, 2 representing the layer degree of freedom treated as a pseudospin. $c^\dagger_{\mathbf{k},\sigma}$ ($c_{\mathbf{k},\sigma}$) is the creation (annihilation) operator for a fermion with momentum $\mathbf{k}$ in layer σ. Assuming, for concreteness, that the gate voltages are such that the Fermi energy lies in the conduction band for BLG ($\sigma = 1$) and in the valence band for SLG ($\sigma = 2$), we have $\varepsilon_{\mathbf{k},1} = -V_g^B + [\hbar^2 v_F^2 k^2 + (\gamma_1^2/2) - [(\gamma_1^4/4) + \hbar^2 v_F^2 k^2 \gamma_1^2]^{1/2}]^{1/2}$ and $\varepsilon_{\mathbf{k},2} = V_g^S - \hbar v_F k$ to which correspond the eigenstates $\psi_{\mathbf{k},1} = (1/\sqrt{2})(1, e^{i\eta m \theta_{\mathbf{k}}})^T$ and $\psi_{\mathbf{k},2} = \frac{1}{\sqrt{2}}(1, -e^{i\eta n \theta_{\mathbf{k}}})^T$, respectively, where $m = 2$ for BLG and $n = 1$ for SLG are the integers that specify the chirality of the two layers and $\eta = +1(-1)$ for states around the K (K') point. Below we consider the states around the K point only as the K' point follows similar analysis; $v_F \approx 10^6$ m/s is the Fermi velocity of SLG close to the Dirac point, $\gamma_1 \approx 400$ meV, and $\theta_{\mathbf{k}} \equiv \arctan(k_y/k_x)$. The form of $\mathcal{H}_0$ that we use is valid as long as $|V_g^S| < 140$ meV and 3 meV $\lesssim |V_g^B| \lesssim 200$ meV [4,32]. For the interacting part of $\mathcal{H}$, we have

$$\begin{aligned}\mathcal{H}_{\rm int} = \frac{1}{2A} \sum_\sigma \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} & V_{\mathbf{q}} f_\sigma(\theta_{\mathbf{k}+\mathbf{q}} - \theta_{\mathbf{k}}) f_\sigma(\theta_{\mathbf{k}'-\mathbf{q}} - \theta_{\mathbf{k}'}) \\ & \times c^\dagger_{\mathbf{k}+\mathbf{q},\sigma} c^\dagger_{\mathbf{k}'-\mathbf{q},\sigma} c_{\mathbf{k}',\sigma} c_{\mathbf{k},\sigma} \\ & + \frac{1}{A} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} V^d_{\mathbf{q}} f_1(\theta_{\mathbf{k}+\mathbf{q}} - \theta_{\mathbf{k}}) f_2(\theta_{\mathbf{k}'-\mathbf{q}} - \theta_{\mathbf{k}'}) \\ & \times c^\dagger_{\mathbf{k}+\mathbf{q},1} c^\dagger_{\mathbf{k}'-\mathbf{q},2} c_{\mathbf{k}',2} c_{\mathbf{k},1}, \end{aligned} \quad (1)$$

where A denotes the area of the heterostructure, $V^d_{\mathbf{q}}$ ($V_{\mathbf{q}}$) refers to the interlayer (intralayer) interaction, and $f_1(\theta_{\mathbf{k}} - \theta_{\mathbf{p}}) = \frac{1}{2}[1 + e^{2i(\theta_{\mathbf{k}} - \theta_{\mathbf{p}})}]$, $f_2(\theta_{\mathbf{k}} - \theta_{\mathbf{p}}) = \frac{1}{2}[1 + e^{i(\theta_{\mathbf{k}} - \theta_{\mathbf{p}})}]$ are factors that arise from the wave function overlap between states $\psi_{\mathbf{k},\sigma}$, $\psi_{\mathbf{p},\sigma'}$.

To decouple the interactions, we use the Hartree-Fock approximation and obtain the mean-field Hamiltonian

$$\mathcal{H}_{\rm MF} = \sum_{\mathbf{k},\sigma,\sigma'} c^\dagger_{\mathbf{k},\sigma} (\Delta^0_{\mathbf{k}} \tau^0_{\sigma\sigma'} - \mathbf{\Delta}_{\mathbf{k}} \cdot \boldsymbol{\tau}_{\sigma\sigma'}) c_{\mathbf{k},\sigma'}, \quad (2)$$

where $[\Delta^0, \mathbf{\Delta} = (\Delta^x, \Delta^y, \Delta^z)]$ are the mean fields and $[\tau^0, \boldsymbol{\tau} = (\tau^x, \tau^y, \tau^z)]$ are the 2×2 identity and Pauli matrices acting in the layer pseudospin space. Because of the asymmetry of the band dispersion between the two layers, the field $\Delta^0_{\mathbf{k}}$ does not vanish, unlike that in symmetric double-layer systems. The transverse components of the pseudospin field $\mathbf{\Delta}_{\mathbf{k}}$ form a complex order parameter $\Delta^\perp_{\mathbf{k}} = \Delta^x_{\mathbf{k}} - i\Delta^y_{\mathbf{k}}$, whose magnitude $|\Delta^\perp_{\mathbf{k}}|$ measures the strength of the particle-hole condensate. The mean fields are given by the following self-consistent equations:

$$\begin{aligned}\Delta^0_{\mathbf{k}} = (\varepsilon_{\mathbf{k},2} + \varepsilon_{\mathbf{k},1})/2 & \\ + \frac{1}{2A} \sum_{\mathbf{p}} & \left[V_{\mathbf{k}-\mathbf{p}} F_1(\theta_{\mathbf{k}-\mathbf{p}}) + \frac{2\pi e^2}{\epsilon} g d \right] (1 - n^-_{\mathbf{p}} - n^+_{\mathbf{p}}) \\ - \frac{1}{2A} \sum_{\mathbf{p}} & V_{\mathbf{k}-\mathbf{p}} F_2(\theta_{\mathbf{k}-\mathbf{p}}) \left[1 + \frac{\Delta^z_{\mathbf{p}}}{E_{\mathbf{p}}} (n^-_{\mathbf{p}} - n^+_{\mathbf{p}}) \right], \end{aligned} \quad (3)$$

$$\begin{aligned}\Delta^z_{\mathbf{k}} = (\varepsilon_{\mathbf{k},2} - \varepsilon_{\mathbf{k},1})/2 & \\ + \frac{1}{2A} \sum_{\mathbf{p}} & \left[V_{\mathbf{k}-\mathbf{p}} F_1(\theta_{\mathbf{k}-\mathbf{p}}) - \frac{2\pi e^2}{\epsilon} g d \right] \\ \times & \left[1 + \frac{\Delta^z_{\mathbf{p}}}{E_{\mathbf{p}}} (n^-_{\mathbf{p}} - n^+_{\mathbf{p}}) \right] \\ - \frac{1}{2A} \sum_{\mathbf{p}} & V_{\mathbf{k}-\mathbf{p}} F_2(\theta_{\mathbf{k}-\mathbf{p}}) (1 - n^-_{\mathbf{p}} - n^+_{\mathbf{p}}), \end{aligned} \quad (4)$$

$$\Delta^\perp_{\mathbf{k}} = \frac{1}{2A} \sum_{\mathbf{p}} V^d_{\mathbf{k}-\mathbf{p}} F^d(\theta_{\mathbf{k}-\mathbf{p}}) \left[\frac{\Delta^\perp_{\mathbf{p}}}{E_{\mathbf{p}}} (n^-_{\mathbf{p}} - n^+_{\mathbf{p}}) \right], \quad (5)$$

where $g = 4$ is the total spin and valley degeneracy and ϵ is the dielectric constant of the embedding media. The $[(2\pi e^2)/\epsilon] g d$ term is specific to the interlayer Coulomb interaction in the direct channel; $n^\pm_{\mathbf{p}} = 1/[\exp(\varepsilon^\pm_{\mathbf{p}}/k_B T) + 1]$ are the occupation numbers at temperature T of the renormalized bands with band energies $\varepsilon^\pm_{\mathbf{k}} = \Delta^0_{\mathbf{k}} \pm E_{\mathbf{k}}$, where $E_{\mathbf{k}} = [(\Delta^z_{\mathbf{k}})^2 + |\Delta^\perp_{\mathbf{k}}|^2]^{1/2}$ and $F_1(\theta_{\mathbf{k}-\mathbf{p}})$, $F_2(\theta_{\mathbf{k}-\mathbf{p}})$, $F^d(\theta_{\mathbf{k}-\mathbf{p}})$ (with $\theta_{\mathbf{k}-\mathbf{p}} \equiv \theta_{\mathbf{k}} - \theta_{\mathbf{p}}$) are angle-dependent chiral factors. Specifically, the intralayer chiral factors have the expressions $F_1(\theta_{\mathbf{k}-\mathbf{p}}) = \frac{1}{4}(\cos 2\theta_{\mathbf{k}-\mathbf{p}} + \cos\theta_{\mathbf{k}-\mathbf{p}} + 2)$ and $F_2(\theta_{\mathbf{k}-\mathbf{p}}) = \frac{1}{4}(\cos 2\theta_{\mathbf{k}-\mathbf{p}} - \cos\theta_{\mathbf{k}-\mathbf{p}})$ for SLG-BLG, whereas the interlayer chiral factor can be written in a general form as

$$F^d(\theta_{\mathbf{k}-\mathbf{p}}) = \tfrac{1}{4}(e^{-in\theta_{\mathbf{k}-\mathbf{p}}} + e^0 + e^{i(m-n)\theta_{\mathbf{k}-\mathbf{p}}} + e^{im\theta_{\mathbf{k}-\mathbf{p}}}). \quad (6)$$

In the SLG-SLG structure, $m = n = 1$, and in the hybrid SLG-BLG structure $m \neq n$ with $m = 2$ and $n = 1$.

To understand the consequence of the difference in the chiral factor $F^d(\theta_{\mathbf{k}-\mathbf{p}})$ on the gap equation between the symmetric SLG-SLG heterostructure and the asymmetric SLG-BLG, let us write the general solution of the gap equation (5) as $\Delta^\perp_{\mathbf{k}} = |\Delta^\perp_{\mathbf{k}}|_J e^{iJ\theta_{\mathbf{k}} + i\phi}$ with the chirality $J = 0, \pm 1, \pm 2, \ldots$ and an arbitrary global phase ϕ. Without loss of generality, we assume $\Delta^0_{\mathbf{k}}$, $\Delta^z_{\mathbf{k}}$, and the magnitude $|\Delta^{\perp}_{\mathbf{k}}|$ to be angle independent (it is straightforward to verify that this assumption is consistent with the self-consistent mean-field equations). The gap equation (5) becomes

$$|\Delta^{\perp}_{\mathbf{k}}|_J = \frac{1}{2A}\sum_{\mathbf{p}} V^d_{\mathbf{k}-\mathbf{p}} F^d(\theta_{\mathbf{k}-\mathbf{p}}) e^{-iJ\theta_{\mathbf{k}-\mathbf{p}}} \times \left[\frac{|\Delta^{\perp}_{\mathbf{p}}|_J}{E_{\mathbf{p}}}(n^-_{\mathbf{p}} - n^+_{\mathbf{p}})\right]. \quad (7)$$

In the case of short-range interactions, $V^d_{\mathbf{k}-\mathbf{p}} = \text{const}$, from Eq. (7) we have that in symmetric systems, such as SLG-SLG, in which $m = n$, for $J = 0$, because of the form of the chiral factor, the effective interaction is twice stronger than that for $J \neq 0$ and therefore the nonchiral $J = 0$ state has a critical temperature higher than that of chiral $J \neq 0$ states. On the contrary, for asymmetric systems in which $m \neq n$, such as SLG-BLG, the chiral factor (6) ensures that the effective interaction is the same for all the four states $J = -n, 0, m - n, m$. As a consequence, for heterostructures such as SLG-BLG in which $m \neq n$, in the presence of short-range interactions, the $J = -n, 0, m - n, m$ states satisfy the same gap equation and, therefore, at the mean-field level, the interlayer phase coherent ground state is fourfold degenerate.

In many cases of interest, we expect that the interactions are not short range but still "central," i.e., depending only on the magnitude $|\mathbf{k} - \mathbf{p}|$. In this case, the parts on the right hand side of Eq. (7) that are odd in $\theta_{\mathbf{k}-\mathbf{p}}$ vanish after integrating over the angle and the gap equation takes the form

$$|\Delta^{\perp}_{\mathbf{k}}|_J = \frac{1}{2A}\sum_{\mathbf{p}} V^d_{\mathbf{k}-\mathbf{p}}\left[\frac{|\Delta^{\perp}_{\mathbf{p}}|_J}{E_{\mathbf{p}}}(n^-_{\mathbf{p}} - n^+_{\mathbf{p}})\right] \times \frac{1}{4}[\cos((n+J)\theta_{\mathbf{k}-\mathbf{p}}) + \cos(J\theta_{\mathbf{k}-\mathbf{p}}) + \cos((m-n-J)\theta_{\mathbf{k}-\mathbf{p}}) + \cos((m-J)\theta_{\mathbf{k}-\mathbf{p}})]. \quad (8)$$

Equation (8) shows that for symmetric heterostructures, i.e., $m = n$, in the case of central interactions the $J = 0$ state again has the highest effective pairing strength and therefore the highest critical temperature [13]. On the other hand, for asymmetric heterostructures in which $m = 2n$ the states $J = 0$ and $J = n$ ($J = 0$ and $J = -n$ for the other valley) have the same and the strongest pairing strength and therefore the ground state is twofold degenerate. Similarly, we find that the free energy is the same for each of the degenerate states.

For the SLG-BLG heterostructure, in the presence of Coulomb interactions, $V^d_{\mathbf{k}-\mathbf{p}} = [(2\pi e^2)/\epsilon][(e^{-|\mathbf{k}-\mathbf{p}|d})/|\mathbf{k}-\mathbf{p}|]$; we therefore find that the ground state is twofold degenerate: around the K (K') point, the nonchiral $J = 0$ interlayer phase coherent state (layer-ferromagnetic state) is degenerate with the chiral $J = 1$ ($J = -1$) state; see Fig. 2. By inversion of the gate voltage V_g, the values of J at the K and K' points are interchanged. We find that the nature, chiral or not chiral, of the ground state strongly affects the dynamical density-density response function for frequencies $\omega \approx 2|\Delta^{\perp}_{\mathbf{k}}|$ (Supplemental Material [33] and Ref. [34]) and therefore that optical measurements should be able to distinguish between the two degenerate states.

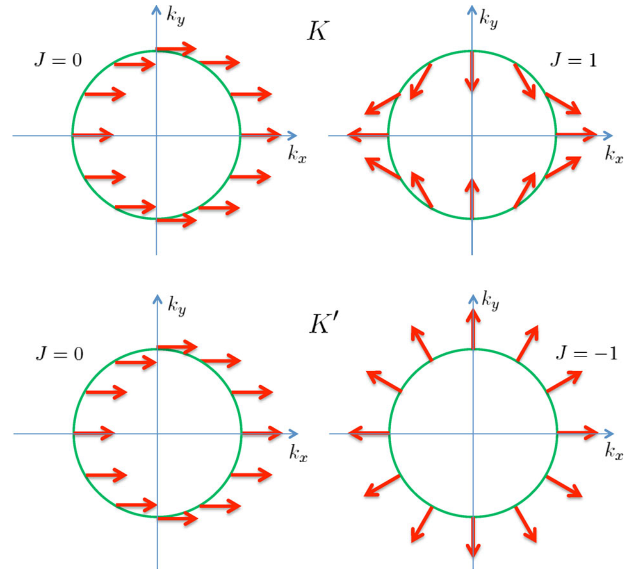

FIG. 2 (color online). Pseudospin configuration on the Fermi surface in the broken-symmetry state for a hybrid SLG-BLG graphene heterostructure around the K point (top) and the K' point (bottom). Here, we have chosen $\phi = 0$. Top: the $J = 0$ state $(\Delta^x_{\mathbf{k}}, \Delta^y_{\mathbf{k}}) = |\Delta^{\perp}_{\mathbf{k}}|(1, 0)$ and the chiral $J = 1$ state $(\Delta^x_{\mathbf{k}}, \Delta^y_{\mathbf{k}}) = |\Delta^{\perp}_{\mathbf{k}}|(\cos\theta_{\mathbf{k}}, -\sin\theta_{\mathbf{k}})$ are degenerate around the K point. Bottom: the $J = 0$ state and the chiral $J = -1$ state $(\Delta^x_{\mathbf{k}}, \Delta^y_{\mathbf{k}}) = |\Delta^{\perp}_{\mathbf{k}}|(\cos\theta_{\mathbf{k}}, +\sin\theta_{\mathbf{k}})$ are degenerate around the K' point.

We emphasize that the degeneracy and chirality of the phase coherent states are due to presence of the chiral factor F^d in the gap equation [Eq. (5)] and do not depend on the details of the band structures of the two layers.

The fact that one of the possible interlayer phase coherent states is chiral opens the possibility to create topologically protected midgap states [30,31,35] at the center of vortices that can be created in the exciton condensate via the axial gauge field [29]. To see this, we observe that we can separate the mean-field Hamiltonian into two parts $\mathcal{H}_{\rm MF} = \mathcal{H}_1 + \mathcal{H}_2$ with $\mathcal{H}_1 = \sum_{\mathbf{k},\sigma,\sigma'} c^{\dagger}_{\mathbf{k},\sigma}(\Delta^0_{\mathbf{k}}\tau^0_{\sigma\sigma'})c_{\mathbf{k},\sigma'}$, $\mathcal{H}_2 = -\sum_{\mathbf{k},\sigma,\sigma'} c^{\dagger}_{\mathbf{k},\sigma}(\boldsymbol{\Delta}_{\mathbf{k}} \cdot \boldsymbol{\tau}_{\sigma\sigma'})c_{\mathbf{k},\sigma'}$. Since $\mathcal{H}_1$ and $\mathcal{H}_2$ commute, the eigenvalues of $\mathcal{H}_{\rm MF}$ are given by the sum of the eigenvalues of $\mathcal{H}_1$ and $\mathcal{H}_2$. $\mathcal{H}_2$ has a symmetric spectrum $\{\pm E_{\mathbf{k}}\}$ that in the chiral $J = 1$ state, due to the $p_x - ip_y$ structure of the order parameter, in the presence of a vortex in the exciton condensate, guarantees the existence of topologically protected midgap states bound to the vortex with energy $\Delta^0_{\mathbf{k}}$ [30,35,36].

From Fig. 3, we see that at $T = 0$ for typical parameter values the peak value $\Delta \equiv |\Delta^{\perp}_{\mathbf{k}}|_{\max}$ of the order parameter magnitude is $\approx 0.075\gamma_1 = 30$ meV. Figure 4(a) shows the

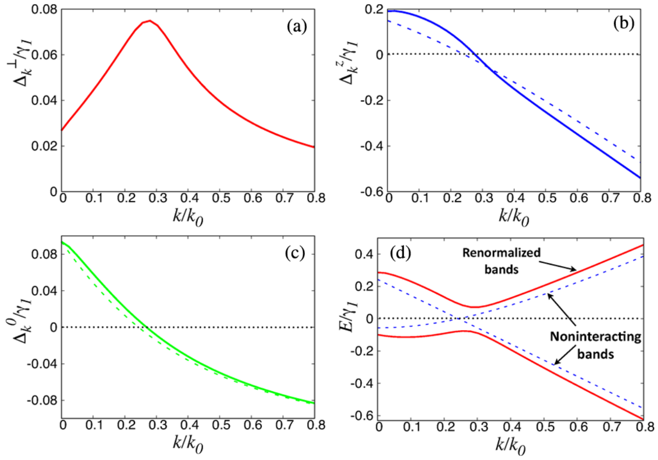

FIG. 3 (color online). Parts (a)–(c) show $|\Delta^{\perp}_{\mathbf{k}}|$, $\Delta^{z}_{\mathbf{k}}$, $\Delta^{0}_{\mathbf{k}}$, respectively, as a function of k [$k_0 \equiv \gamma_1/(\hbar v_F)$], for $T = 0$, $d = 1$ nm, $\alpha = 1$, and $V_g = 0.3\gamma_1$. In (b) and (c), the dashed lines show $\Delta^{z}_{\mathbf{k}}$ and $\Delta^{0}_{\mathbf{k}}$ respectively in the noninteracting case. (d) The solid (dashed) lines show the renormalized (noninteracting) bands.

dependence of Δ on V_g for both SLG-BLG and SLG-SLG at $d = 1$ nm and $\alpha = 1$, where $\alpha \equiv e^2/(\epsilon\hbar v_F)$. We find that at low bias ($V_g/d < 60$ meV/nm) Δ is larger in the hybrid SLG-BLG heterostructures. Compared to the symmetric SLG-SLG structure, in the SLG-BLG structure the density of states (DOS) in one of the layers (BLG) is higher than in the SLG-SLG structure, and the interlayer chiral factor $F^d(\theta_{\mathbf{k}-\mathbf{p}})$ oscillates more rapidly. The first effect favors the formation of the exciton condensate and therefore enhances Δ whereas the second effect tends to suppress it. We can then understand the scaling with V_g of the ratio (Δ_ρ) between Δ for SLG-BLG and for SLG-SLG [inset of Fig. 4(a)] as a result of the competition of two effects: the DOS effect dominates at low V_g and the fast oscillation of $F^d(\theta_{\mathbf{k}-\mathbf{p}})$ takes over at high V_g. Figure 4(b) also shows that in the weak coupling regime ($\alpha < 1$) the interlayer coherence can be stronger in SLG-BLG than in SLG-SLG.

The value of Δ, for typical values of $V_g \approx 0.3\gamma_1$, suggests a mean-field critical temperature $T_c \lesssim 300$ K. This

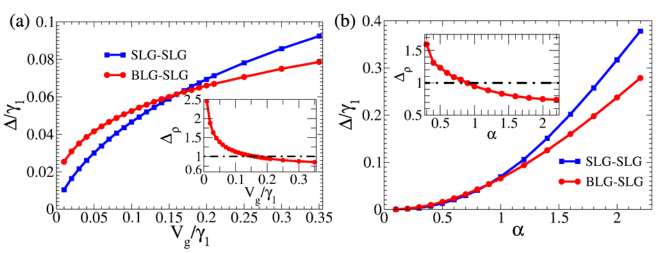

FIG. 4 (color online). $\Delta \equiv |\Delta^{\perp}_{\mathbf{k}}|_{\max}$ as a function of V_g (a) and α (b) in the hybrid SLG-BLG structure and the symmetric SLG-SLG structure for $T = 0$ and $d = 1$ nm. In (a), $\alpha = 1$, and in (b), $V_g = 0.2\gamma_1$. The insets show the ratio (Δ_ρ) between Δ in SLG-BLG and Δ in SLG-SLG.

value is an overestimate. Because the system is two dimensional and the broken symmetry is U(1), T_c is reduced to the Berezinskii-Kosterlitz-Thouless temperature ($T_{\rm BKT}$), above which we have the proliferation of unbound vortices and antivortices of the condensate. In addition, thermal and quantum phase fluctuations [37], screening [16,38–46], and disorder [47,48] can considerably reduce T_c. An accurate estimate of T_c is beyond the reach of theory also due to the uncertainties about the experimental conditions. However, the degeneracy and chirality of the ground state are robust and independent of the exact value of T_c. Screening and disorder are expected to be the dominant factors in suppressing T_c [12]. Screening in general will preserve the central nature of the interaction and therefore will not affect the degeneracy and chirality of the phase coherent state. Similarly, the presence of disorder will renormalize the order parameter and therefore T_c but also does not affect our main findings. To show this, let us denote by a tilde the disorder-renormalized fields. For $\tilde{\Delta}^{\perp}_{\mathbf{k}}$ we find

$$\tilde{\Delta}^{\perp}_{\mathbf{k}} = \Delta^{\perp}_{\mathbf{k}} - \frac{n_i}{A}\sum_{\mathbf{p}} \frac{F^d(\theta_{\mathbf{k}-\mathbf{p}})U_1(\mathbf{k}-\mathbf{p})U_2^*(\mathbf{k}-\mathbf{p})\tilde{\Delta}^{\perp}_{\mathbf{p}}}{-(i\omega_n - \tilde{\Delta}^0_{\mathbf{p}})^2 + (\tilde{\Delta}^z_{\mathbf{p}})^2 + |\tilde{\Delta}^{\perp}_{\mathbf{p}}|^2}, \tag{9}$$

where n_i is the impurity density, U_σ is the disorder potential in layer σ, and ω_n are the Matsubara frequencies. Equation (9) shows that the chiral factor F^d appears in the same way as in the gap equation valid in the clean limit. This guarantees that even in the presence of disorder, the chiral and the nonchiral solutions are degenerate, considering that for almost all cases of interest $U_\sigma(\theta_{\mathbf{k}-\mathbf{p}}) = U_\sigma(-\theta_{\mathbf{k}-\mathbf{p}})$.

Considering that we find that in SLG-BLG the mean-field T_c value for unscreened Coulomb interaction is of the same order as that in SLG-SLG and that screening, disorder, thermal, and quantum fluctuations are expected to affect T_c similarly in the two systems, we conclude that in realistic setups T_c for SLG-BLG should be of the same order as that for SLG-SLG. Recent results [12] show hints of an exciton condensate for SLG-SLG in current experimental conditions. We can then conclude that the combined effects of screening and disorder in SLG-SLG and SLG-BLG heterostructures might suppress T_c but should not prevent the experimental observation of the predicted interlayer phase coherent states.

In conclusion, we have shown that in hybrid heterostructures in which the electrons in different layers have different chirality (m in one layer and n in the other) the interlayer phase coherent state is fourfold degenerate for short-range interactions and twofold degenerate for long-range central interactions when $m = 2n$. Moreover, we find that one of the degenerate states is always a chiral superfluid state, a fact that implies the presence of protected midgap states in the presence of vortices in the exciton condensate. We also find that these properties of the ground state are robust and are not affected by effects such as screening and disorder that on the other hand can strongly suppress T_c for the formation of the interlayer phase coherent state.

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